Consistent Return Estimates
-
The Black-Litterman Approach

Frankfurt MathFinance Workshop 2005

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PM High Alpha / Portfolio Construction

- April 2005 -

Abstract

The Black-Litterman Approach

Consistent asset return estimates - saving classical mean/variance…

In asset management, the forecast of asset returns is essential within the investment process. In this context, the Black-Litterman approach (1992) yields consistent asset return forecasts as a weighted combination of (strategic) market equilibrium returns and (tactical) subjective forecasts (”views”). The Black-Litterman formalism allows to implement both absolute views (return levels) and relative views (outperforming vs. underperforming assets) for selected assets investigated under „core competence“. For any particular view, individual confidence levels for the return estimates have to be specified. The formalism spreads these informations consistently across all assets in the portfolio. The BL-revised returns then serve as a consistent input for mean-variance portfolio optimisation procedures, thus allowing for the implementation of additional constraints. It turns out that BL-optimized portfolios overcome some well-known Markowitz insufficiencies as unrealistic sensitivity to input factors or extreme portfolio weights. The BL process will be introduced both from its theoretical background and its implementation in practice.

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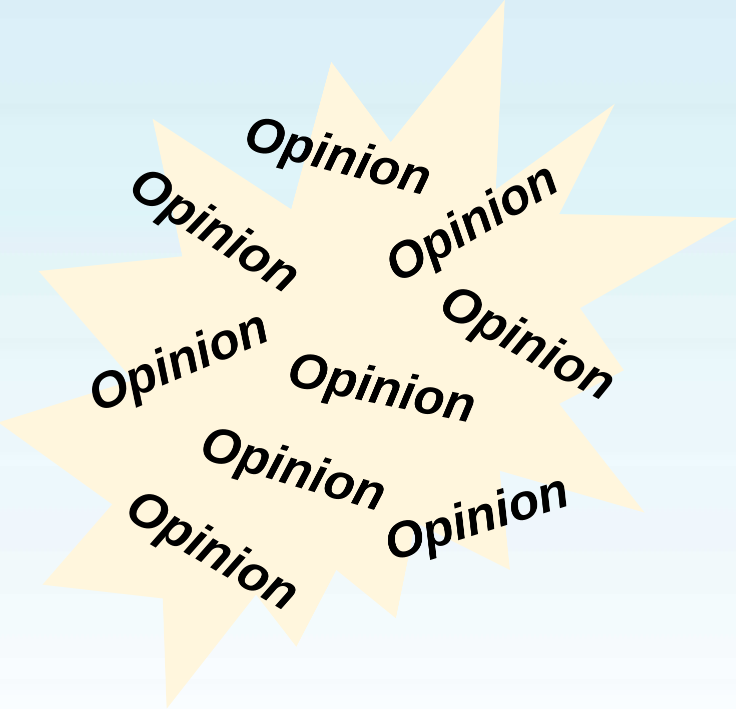
Investment Process: Opinions & Portfolio Context

RESEARCH

- The c.p. world of core competences -

PORTFOLIO CONSTRUCTION

- The portfolio context: Thinking in terms of correlations -



Opinion
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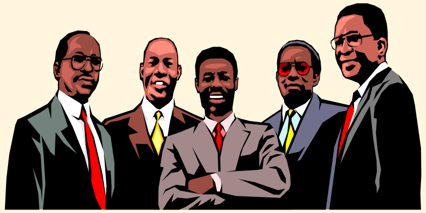
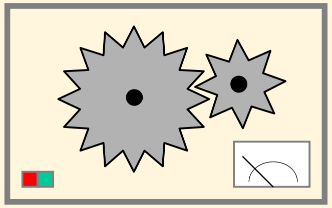
?

Committee



or

Quant tool



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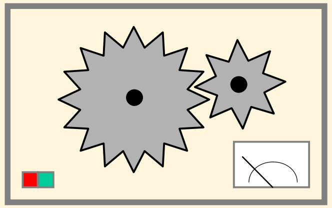
c.p. = ceteribus paribus

Defining the subjectives… ⇒ Black & Litterman

Let‘s talk about the ... Quant tool



Some nice-to-haves

„Consistent“ (non-c.p.) input for portfolio construction

Intuitive results

Tactical deviations from „some“ strategic allocation

Transparent

Managable

Reliable output

Weighting estimates according to confidence

Overcome some of the problems of plain MV (Markowitz)

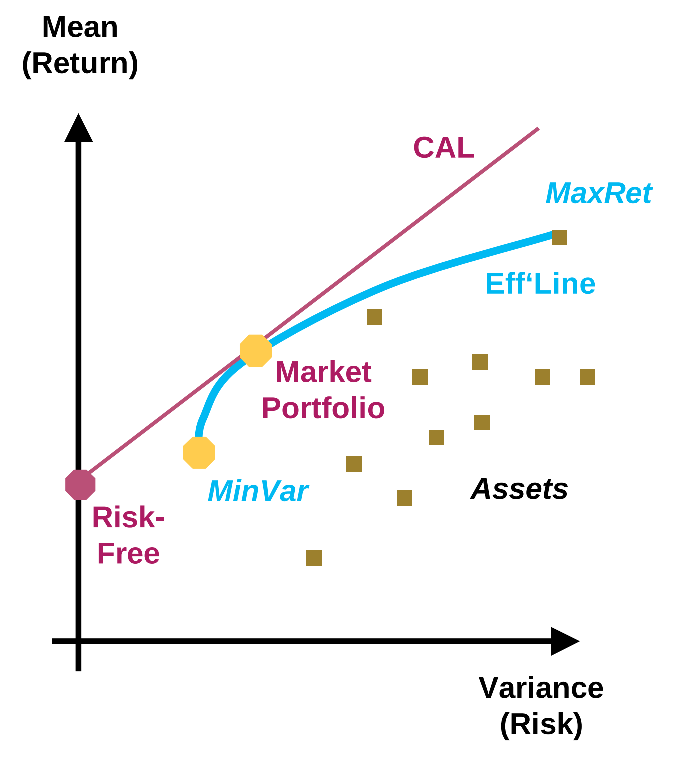
Black-Litterman

Topics to be discussed…

- Classical Markowitz - the straight way
- MV-optimized portfolios
- BL - the role within the investment process
- BL - implementation
- BL - example

The Markowitz Approach - in short

Efficient portfolios in the mean-variance approach



- **Starting point** in a world of normally distributed returns: The assets are described by the first two moments of return - mean and variance.
- In an **efficient portfolio** the assets are weighted such that for any given level of risk a maximum return is achieved. (equivalently: for a given return the risk is minimized). Diversification reduces risk.
- All efficient portfolios form the **efficiency line**. It starts in the minimum variance portfolio and ends in the maximum return portfolio (which is the asset of maximum return).
- If a risk-free asset exists, all efficient portfolios are located on the **Capital Allocation Line** (CAL), starting at the risk-free asset and tangentially touching the efficiency line at the market portfolio. Efficient portfolios are then a combination of the risky **market portfolio and the risk-free asset** (with risk-free *long* or *short*), a.k.a. *Tobin Separation* (*1958*).

The Markowitz Approach - dealing with its problems

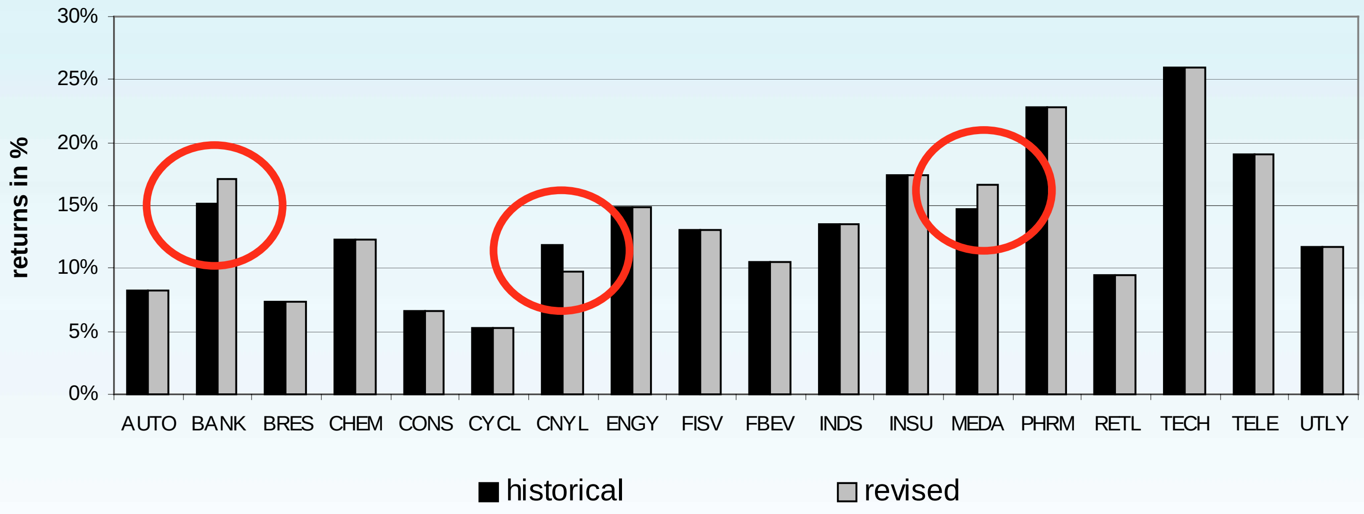
Deficits of the mean-variance (MV) concept, suggestions for solutions...

Deficit	**Possible Solution**
High sensitivity on inputs (return estimates!) leads to large weight fluctuations in the optimal portfolio.	***Black-Litterman***
„Corner solutions“: Extreme portfolio weights (also in the case of optimization algorithms using constraints)	***Black-Litterman***
Aggregation: Consistent aggregation of huge number of estimated returns overburdens the investment process	***Black-Litterman***
No quantification of confidence in estimated returns	***Black-Litterman***
One-periodical approach	Multi-period approaches, ...
„Variance“= restricting risk to symmetric return volatility	VaR, ...
Requires ex-ante-estimates of covariance matrix	Vola-modeling, ...

The Markowitz Approach - straight is not enough

When return estimates change...

- Let the investment universe be the 18 STOXX sectors in Euroland.
- **At time 0**: Returns : Historical returns
  Weights : To be determined via MV
- **At time 1**: Returns : +2%pts for BANK and MEDA, -2%pts for CNYL, others unchanged
  Weights : To be determined via MV



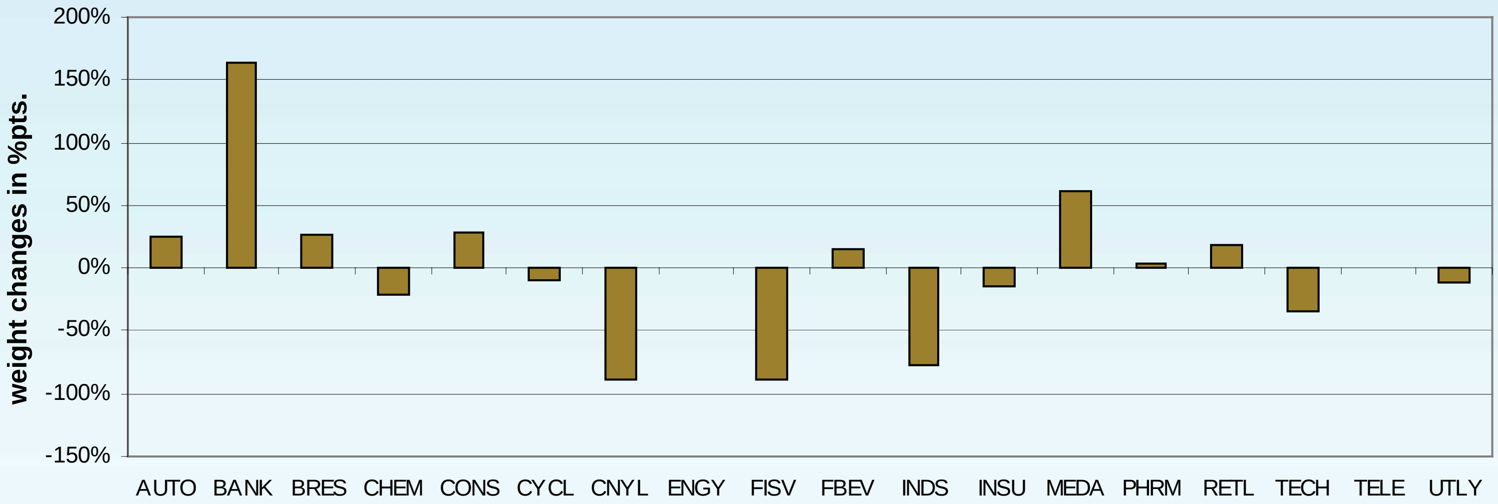
Historical returns and revised expected returns

The Markowitz Approach - straight is not enough

Fluctuation of asset weights due to revised return estimates

- Even small and selected changes in expected returns lead to huge unrealistic shifts in asset weights! (γ=3, historical covariances)

Changes in portfolio weights due to changes of expected returns



- Problems: Communication of results, (re-) allocation in real portfolios, acceptance of method.

The Markowitz Approach - straight

The formal optimization approach for risky assets, basic outline.

- Markowitz Theory relates risk & return

- MV optimization problem:

$$w^{\mathrm{T}} R - \frac{\gamma}{2} \cdot w^{\mathrm{T}} \Omega \, w \rightarrow \max_{w}$$

R = *vector of returns*
Ω = *covariance matrix*
γ = *risk aversion parameter*
w = *vector of weights*

- Solution: Optimal portfolio weights w^* (no constraints):

$$w^* = (\gamma \, \Omega)^{-1} R$$

- Markowitz provides a mechanism to achieve optimal (efficient) portfolios.
  What about the input factors ???

Extending the Markowitz Approach

Equilibrium returns

Supply & demand

- Traditional approach of maximum return & minimum risk is demand-side perspective.
- Need to balance with supply-side...

Concept of equilibrium returns:

- The market portfolio exists in market equilibrium, i.e. supply & demand are in equilibrium.
- Therefore, equilibrium returns reflect neutral „fair“ reference returns Π :
- Inverse optimization yields:

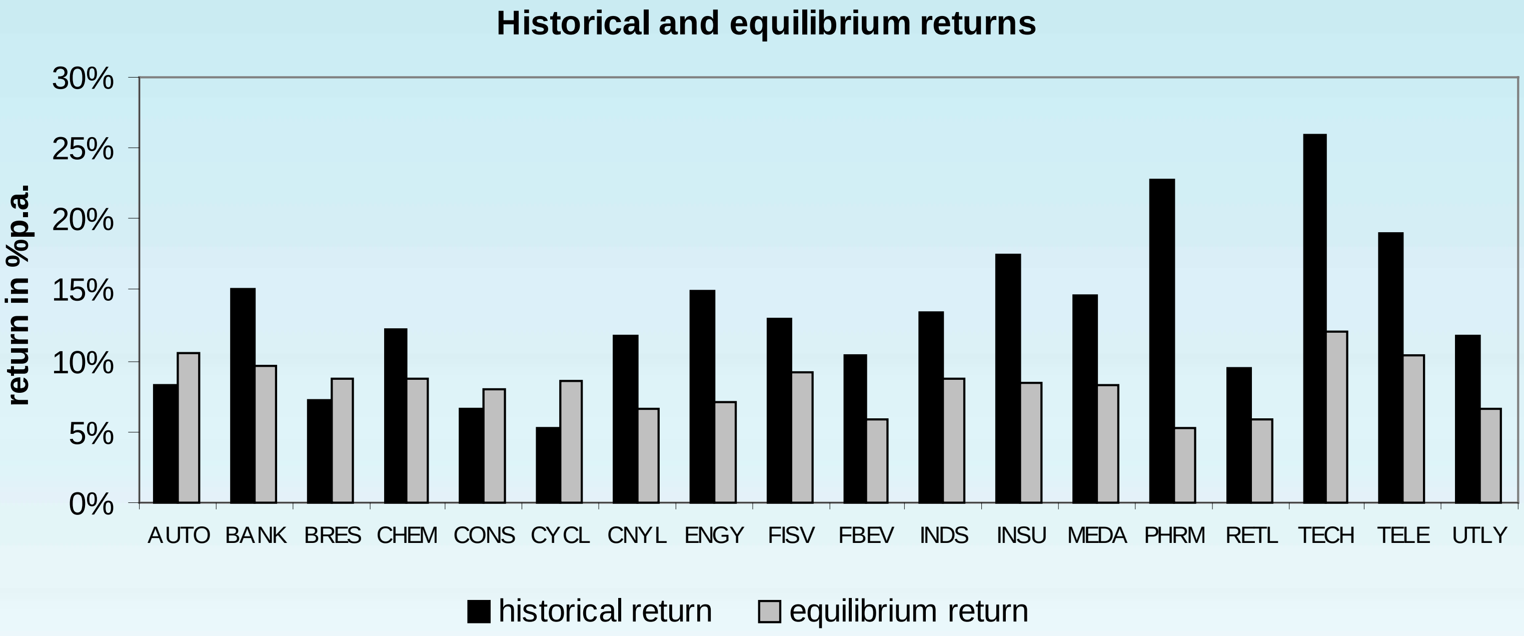
$$\Pi = (\gamma\,\Omega)\, w_{\mathrm{MCap}} \qquad w_{MCap} = market\ capitalization$$

Conclusion

- Use of equilibrium returns as a long term strategic reference for any return estimate („market neutral starting point“).

Extending the Markowitz Approach

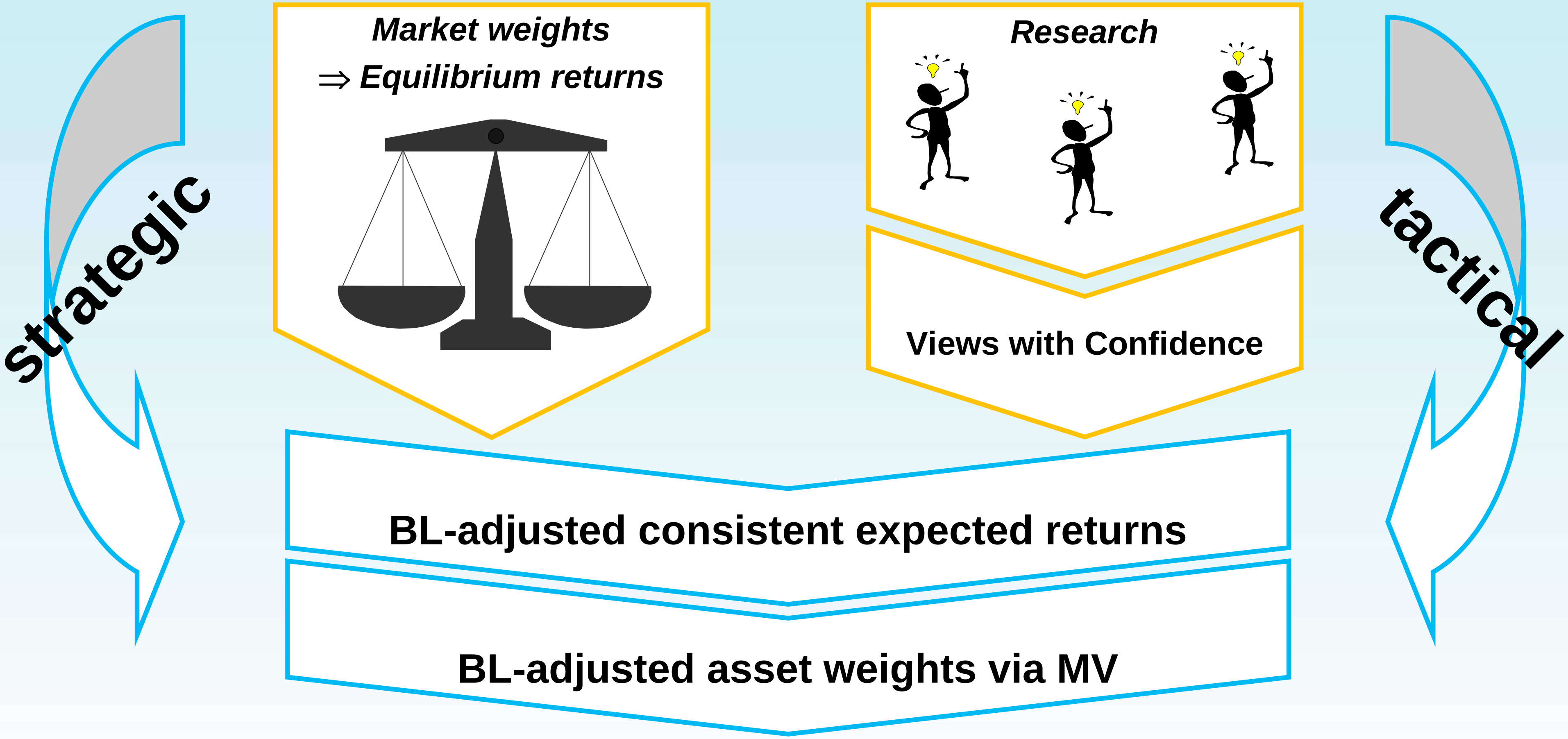
Equilibrium (or implicit) returns for the STOXX sectors



- These implicit returns serve as reference returns for all further investigations
- Note that equilibrium returns are *calculated*; they do not require any estimation procedure.

Black-Litterman Approach - basic outline

BL provides a concept to combine long term equilibrium returns with short term return estimates as a consistent input for MV-optimization.



strategic

Market weights

⇒ Equilibrium returns

Research

Views with Confidence

tactical

BL-adjusted consistent expected returns

BL-adjusted asset weights via MV

Black-Litterman Approach - going math

Optimization s.t. contraints

- Determine the optimal estimate E(R), which minimizes the variance of E(R) w.r.t. equilibrium returns Π: (minimizing the Mahalanobis distance):

$$[E(R)-\Pi]^{\mathrm{T}} \cdot (\tau\,\Omega)^{-1} \cdot [E(R)-\Pi] \rightarrow \min_{E(R)}$$

where: $E(R) = \Pi + \nu \quad \text{with} \quad \nu \sim N(0, \tau\,\Omega).$

s.t.

$$P \cdot E(R) = \begin{cases} V & \text{certain Views} \\ V + e & \text{uncertain Views} \end{cases}$$

where: $P \cdot E(R) \sim N(V, \Sigma)\,,\ \Sigma_{ii} = e_i$

Black-Litterman Approach - the formulas

Master equations for the BL-return estimates

- Solution in the case of **certain estimates** ($\Sigma \equiv$ zero matrix):

$$\overline{E}(R) = \Pi + (\tau\,\Omega)\,P^{\mathrm{T}} \cdot \left(P(\tau\,\Omega)\,P^{\mathrm{T}}\right)^{-1} \cdot \left(V - P\,\Pi\right)$$

- Solution in the case of **uncertain estimates** (Σ = diagonal matrix):

$$\overline{E}(R) = \left[(\tau\,\Omega)^{-1} + P^{\mathrm{T}}\Sigma^{-1}P\right]^{-1} \cdot \left[(\tau\,\Omega)^{-1}\Pi + P^{\mathrm{T}}\Sigma^{-1}V\right]$$

- The **constraints** $P \cdot E(R) = V + e$ are implicitly fulfilled.

Black-Litterman Approach - more math I

Formal proof for the case „certain estimates“

Proposition: The optimization problem $\left[E(R)-\Pi\right]^{\mathrm{T}}\cdot(\tau\,\Omega)^{-1}\cdot\left[E(R)-\Pi\right]\rightarrow\min\limits_{E(R)}$ s.t. $P\cdot E(R)=V$

yields variance-minimum returns $\overline{E}(R)=\Pi+(\tau\,\Omega)\,P^{\mathrm{T}}\cdot\left(P(\tau\,\Omega)\,P^{\mathrm{T}}\right)^{-1}\cdot\left(V-P\,\Pi\right)$

Proof:

Lagrangian: $L:=\left[E-\Pi\right]^{\mathrm{T}}\cdot(\tau\,\Omega)^{-1}\cdot\left[E-\Pi\right]\text{ - }\lambda\cdot(PE\text{-}V)$

f.o.c.'s: (1) $\frac{\partial L}{\partial E}=0$ and (2) $\frac{\partial L}{\partial \lambda}=0$

"scalarizing": $$\frac{\partial L}{\partial E_i}=\frac{\partial}{\partial E_i}\left\{\tau^{-1}\sum_{j,k}E_j\Omega^{-1}_{jk}E_k-\sum_k\lambda_k\left(\sum_j P_{kj}E_j-V_k\right)\right\}=2\tau^{-1}\sum_k\Omega^{-1}_{ik}E_k-\sum_k P_{ki}\lambda_k$$

$$\frac{\partial L}{\partial \lambda_i}=-\frac{\partial}{\partial \lambda_i}\sum_k\lambda_k\left(\sum_j P_{kj}E_j-V_k\right)=-\left(\sum_j P_{ij}E_j-V_i\right)$$

"*re*vectorizing": (1) $\frac{\partial L}{\partial E}=2(\tau\,\Omega)^{-1}\,E-2(\tau\,\Omega)^{-1}\,\Pi-P\lambda=0$ and (2) $\frac{\partial L}{\partial \lambda}=PE-V\;=0$

Solve Eq. (1) for E, insert in Eq.(2), thereof expression for λ, result follows with Eq. (1). ◊

Black-Litterman Approach - more math II

Formal proof for the case „uncertain estimates“ (BL-Master Formula)

Proposition: $[E(R)-\Pi]^{\mathrm{T}} \cdot (\tau\,\Omega)^{-1} \cdot [E(R)-\Pi] \rightarrow \min\limits_{E(R)}$ with constraints $P \cdot E(R) = V + e$

yields variance-minimum returns $\overline{E}(R) = \left[(\tau\,\Omega)^{-1} + P^{\mathrm{T}}\Sigma^{-1}P\right]^{-1} \cdot \left[(\tau\,\Omega)^{-1}\Pi + P^{\mathrm{T}}\Sigma^{-1}V\right]$

Proof:

$$\text{Given}: \ \Pi = E(R) + v \quad \text{and} \quad V = P \cdot E(R) + e$$

$$\text{Setting} \quad \mathrm{Y} := \begin{pmatrix} \Pi \\ \mathrm{V} \end{pmatrix}, \quad X := \begin{pmatrix} I \\ P^T \end{pmatrix}, \quad W := \begin{pmatrix} \tau\Omega & 0 \\ 0 & \Sigma \end{pmatrix} \quad \text{and} \quad \mathrm{u} \sim N(0, W)$$

$$\text{so that} \quad Y = X \cdot E(R) + u \quad \text{and using generalized least square} \quad E(R) = \left(X^T W^{-1} X\right)^{-1} X^T W^{-1} Y \quad \text{we get}$$

$$E(R) = \left[\begin{pmatrix} I & P^T \end{pmatrix} \begin{pmatrix} \tau\Omega & 0 \\ 0 & \Sigma \end{pmatrix}^{-1} \begin{pmatrix} I \\ P \end{pmatrix} \right]^{-1} \times \left[\begin{pmatrix} I & P^T \end{pmatrix} \begin{pmatrix} \tau\Omega & 0 \\ 0 & \Sigma \end{pmatrix}^{-1} \begin{pmatrix} \Pi \\ V \end{pmatrix} \right]$$

$$= \left[\begin{pmatrix} (\tau\Omega)^{-1} & P^T\Sigma^{-1} \end{pmatrix} \begin{pmatrix} I \\ P \end{pmatrix} \right]^{-1} \times \left[\begin{pmatrix} (\tau\Omega)^{-1} & P^T\Sigma^{-1} \end{pmatrix} \begin{pmatrix} \Pi \\ V \end{pmatrix} \right] = \left[(\tau\Omega)^{-1} + P^T\Sigma^{-1}P\right]^{-1} \times \left[(\tau\Omega)^{-1}\Pi + P^T\Sigma^{-1}V\right] \Diamond$$

Non-Bayesian proof, taken from *„Asset Allocation Model“, Daniel Blamont, Global Markets Research, Dt.Bank, July 30 2003*

For Bayesian proof see, *e.g., Fusai and Meucci*

Black-Littermann - the master formula

$$E(R) = \left[(\tau\,\Omega)^{-1} + P^{\mathrm{T}}\Sigma^{-1}P\right]^{-1} \cdot \left[(\tau\,\Omega)^{-1} \cdot \Pi + P^{\mathrm{T}}\Sigma^{-1}\underbrace{P \bullet P^{-1}}_{=\mathbb{1}}V\right]$$

- Complex interaction between equilibrium returns and subjective return expectations
- First factor („Denominator“) : Normalisation
- Second factor („Numerator“) : Balance between Π (= equilibrium returns) and **V** (= Views). Inverse of covariance $(\tau\,\Omega)^{-1}$ and confidence $P^{T}\Sigma^{-1}P$ serve as weighting factors.
- Constituents: Matrix $\tau\,\Omega$: covariance of historical returns, $\tau =$ parameter
  Matrix *P* : formal aggregation of Views
  Matrix Σ^{-1} : confidence in Views (Σ = „covariance of estimated Views“)
  Σ assumed to be diagonal, i.e. no cross-informations on Views.
- Limiting case “no estimates“ $\Leftrightarrow P=0$: $E(R) = \Pi$ i.e. BL-returns = equil. returns.
- Limiting case “no estimation errors“ $\Leftrightarrow \Sigma^{-1} \rightarrow \infty$: $E(R) = P^{-1}V$ i.e. BL-returns = View returns.

Black-Litterman Approach - remarks

Use of CAPM to determine equilibrium returns

- Alternative to inverse optimization: Equilibrium returns Π_{Eq} from CAPM
- Additional input for CAPM: Risk-free rate r_f, risk premium vs market (M), Beta coefficients
- Evaluation:

  (fair return for asset i) $\Pi_{i,Eq} = r_f + \beta_i \cdot \left(r_M - r_f\right)$ with $\beta_i = \dfrac{\sigma_{iM}}{\sigma_M^2}$

BL returns as a Bayesian a-posteriori estimator

- Bayes Theorem (or "Law" or "Rule") states how to determine conditional expectations.
- Given an a-priori known distribution of a random variable. Adding new information leads to a revised conditional distribution, the so-called a-posteriori distribution (result of „learning").
- BL-return estimates are a-posteriori (multivariate) normally distributed return expectations.

Black-Litterman Approach - γ

Remark: Risk aversion parameter γ

- How does risk change for an additional bp of return?
- **Suggestions:**

Satchell & Scowcroft and Best & Grauer:

Let $\gamma = (r_M - r_f) / \sigma_M^2$

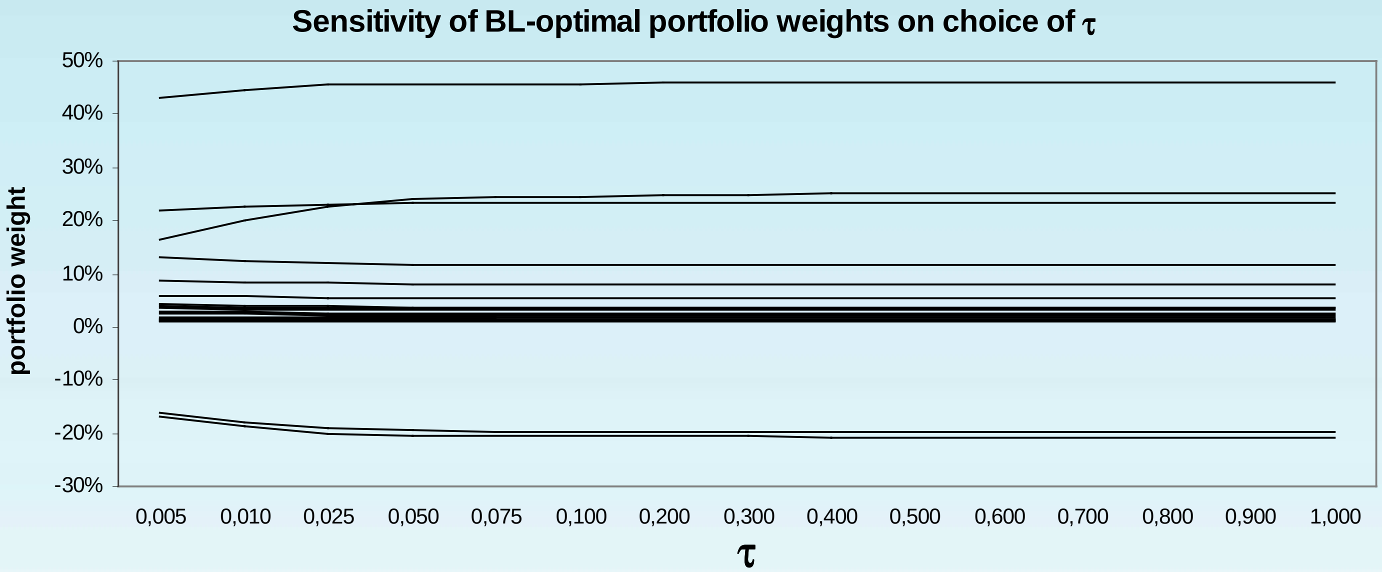
where $\sigma_M^2 = w^T \Omega w$, w = market cap.

Zimmermann et al.: σ_M=16.9% p.a. for STOXX-data (own calculation)
Chose $\gamma = 3$, which corresponds to a risk premium of 8.6%.

Idzorek: (DJIA, USA): Risk premium=7.5%: $\gamma = 2.25$.

Black-Litterman Approach - τ

Remark: Skaling parameter τ for the covariance matrix



Results/Setting

- Covariances of expected returns are proportional to historical covariances: $\tau\ \Omega$.
- τ measures confidence in benchmark, i.e. overall balance between BM and Views.
- τ small: *VAR[E(R)] << VAR[historical returns]*
- $\tau = 0.3$ “plausible“ (used for numerical evaluations throughout).

Black-Litterman Approach - some real problems

Additional remarks on the recent remarks

- Calibration problems with parameter τ („plausible“, „adjusted to IR=1“, ...)
- Calibration problems with parameter γ („world wide risk aversion“, ...)
- Calibration problems with expressing the degree of confidence („1..3“, „0..100%“)

Black-Litterman Approach - Views

Implementing Views on expected returns deviating from equilibrium figures

Views

- Return estimates differing from the (strategic) equilibrium returns are the essential input to the BL estimation process.

Specification of Views

- ... as absolute return expectations for individual assets

  and / or

- ... as relative return expectations relating assets or aggregates of assets.

  Formal constraint: #Views $\leq$ #Assets.

Confidence

- Each View has to be assigned the level of confidence for an interval of uncertainty.

Selective Views

- Views can be restricted to selected assets for which in-depth analysis is available.

Black-Litterman Approach - Views

- A **relative View** can be stated as follows: „The sectors Pharmacy and Industry will outperform Telecom and Technology by 3% ± 1% with a confidence of 90%“:

$$\begin{aligned}&\left[w_{PHRM}\cdot E(R_{PHRM})+w_{INDU}\cdot E(R_{INDU})\right]\\&\qquad-\left[w_{TELE}\cdot E(R_{TELE})+w_{TECH}\cdot E(R_{TECH})\right]=3\%+(0.61\%)^2\end{aligned}$$

- Basically: A *long*-portfolio with outperformers, a *short*-portfolio with underperformers.

- An **absolute View** can be stated as follows: „The sector of Non-Cyclical Goods (CNYL) will perform better than stated by the equilibrium return of 6.66%. Our new target return is 7.5% with 90% of confidence within a range of ±1.5%“:

$$1\cdot E(R_{CNYL})=7.5\%+(0.91\%)^2$$

Black-Litterman Approach - combining Views I

Formal aggregation of Views

- Relative and absolute Views are aggregated in a system of linear equations:

$$P \cdot E(R) = V + e$$

where (k = #Views and n = #Assets, with k $\le$ n):

E(R) = n$\times$1 vector of expected asset returns, unknown
P = k$\times$n matrix, weighting the asses
V = k$\times$1 vector, absolute / relative return expectations (i.e., levels or over-/underperforming)
e = k$\times$1 vector of squared StDev‘s
(note that Σ^{-1} is a kxk diagonal matrix expressing confidence (assuming independent estimation errors) with $\Sigma_{ii} = e_i$)

- This relation is incorporated in the BL master equation.

Black-Litterman Approach - combining Views II

Result shown from the example mentioned on page 24.

- Combining the aforementioned Views using $P \cdot E = V + e$, we get:

$$P \cdot \begin{pmatrix} E(R_{AUTO}) \\ \vdots \\ E(R_{UTLY}) \end{pmatrix} = \begin{pmatrix} 3\% \\ 7.5\% \end{pmatrix} + \begin{pmatrix} (0.61\%)^2 \\ (0.91\%)^2 \end{pmatrix}$$

- The View-related weights P are given by:

long positions

short positions

$$P = \begin{pmatrix} View\,1, rel. \\ View\,2, abs. \end{pmatrix} = \begin{pmatrix} 0 & \cdots 0 & \cdots & 0 & 0.34 & 0 & 0 & 0.66 & 0 & -0.51 & -0.49 & 0 \\ 0 \; \cdots & 1 \; 0 & & & \cdots & & \cdots & & & \cdots & & 0 \end{pmatrix}$$

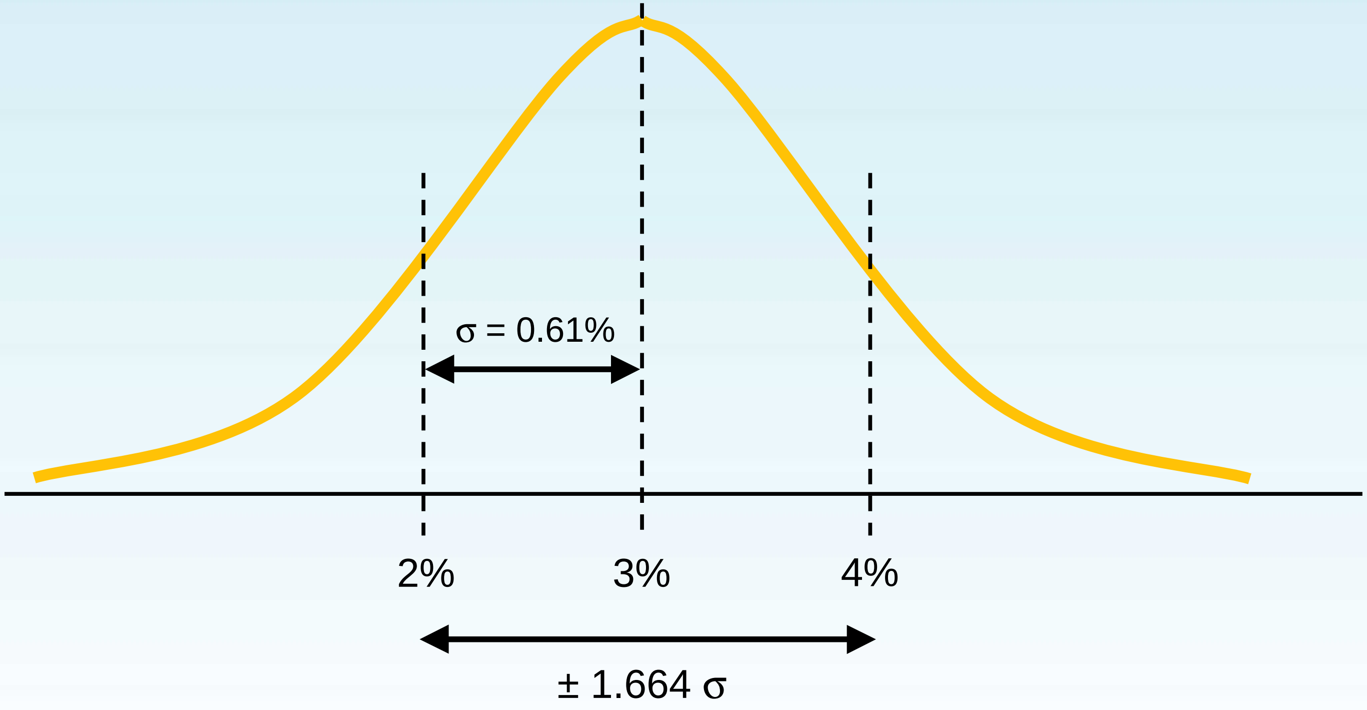
AUTO ⋮ CNYL ENGY FISV FBEV INDS INSU MEDA PHRM RETL TECH TELE UTLY

asset projector

Black-Litterman Approach - confidence

Technical note on „confidence“

- Comment on determination of *e* : The fact that the amount of, e.g., *relative* outperformance (View 1) of 3% ±1% is assigned a 90% probability is interpreted within a normal distribution.
- ***mean* = 3%** and ***variance* = *VAR* = σ^2 = (0.61%)^2 = $e_1 \equiv \Sigma_{11}$.**



Black-Litterman Approach - example in detail

„Sector allocation, Dow Jones STOXX“

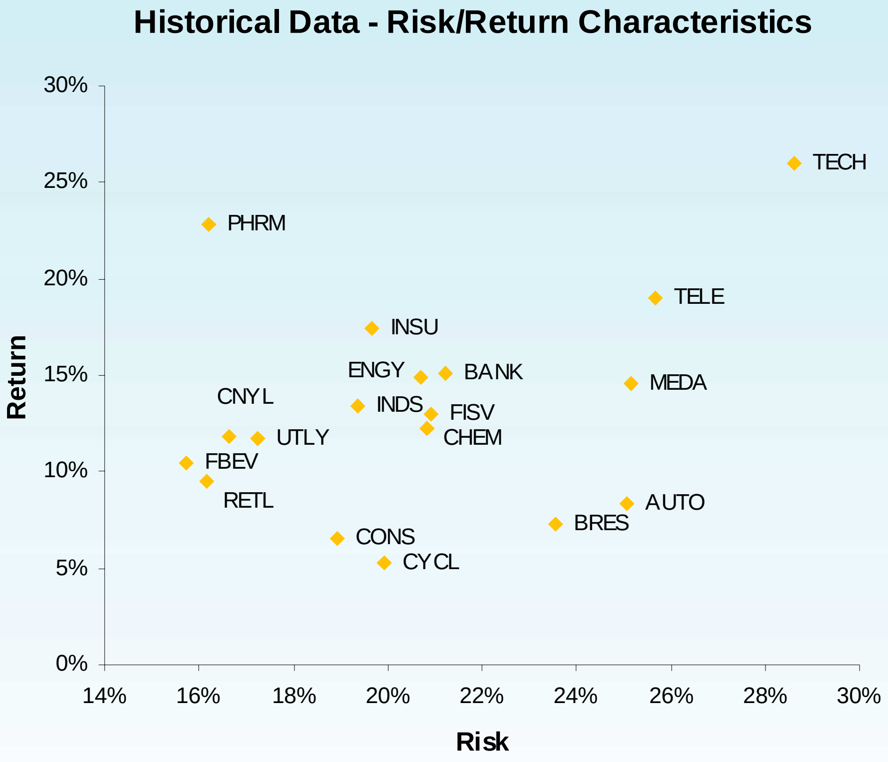
- Example follows the lines of
  „Einsatz des Black-Litterman-Verfahrens in der Asset Allocation“, *H.Zimmermann et al.*
  publ. in „Handbuch Asset Allocation“
  (Editors: Dichtl, Schlenger u. Kleeberg, publ. by Uhlenbruch-Verlag, 2002).
- Notation, scenarios and data therein have been used, some data were missing.
- Missing data - volatilities and covariances - had to be calculated from scratch, thus causing some deviations in the numerical results between this presentation and cited literature.
  Nevertheless, all relevant results are reproduced.
- All calculations can be (have been) implemented and performed in Excel (TM).

Black-Litterman Approach - the data

Sectors in the Dow Jones STOXX index

- Monthly returns in Sfr (Swiss francs), period: 06/1993 - 11/2000, annualized data.

Sector	hist.Return	hist.Volatility	MarketCap
total: **18**	**average:** **16,22%**		**total:** **100,01%**
AUTO	8,32%	25,09%	1,65%
BANK	15,14%	21,21%	15,04%
BRES	7,31%	23,56%	1,22%
CHEM	12,25%	20,81%	1,80%
CONS	6,56%	18,92%	1,26%
CYCL	5,24%	19,94%	2,85%
CNYL	11,80%	16,66%	2,90%
ENGY	14,92%	20,72%	10,30%
FISV	13,01%	20,91%	4,12%
FBEV	10,47%	15,72%	4,59%
INDS	13,45%	19,35%	5,19%
INSU	17,43%	19,68%	6,89%
MEDA	14,63%	25,17%	3,27%
PHRM	22,83%	16,20%	10,24%
RETL	9,49%	16,16%	2,27%
TECH	25,95%	28,60%	11,03%
TELE	18,99%	25,69%	10,56%
UTLY	11,77%	17,25%	4,83%



Black-Litterman Approach - the correlations

Correlation matrix of Dow Jones STOXX sectors

- Calculation based on monthly returns

	AUTO	BANK	BRES	CHEM	CONS	CYCL	CNYL	ENGY	FISV	FBEV	INDS	INSU	MEDA	PHRM	RETL	TECH	TELE	UTLY
AUTO	**100%**	**74%**	**73%**	**83%**	**78%**	**75%**	**73%**	55%	**73%**	**71%**	**79%**	**72%**	46%	43%	68%	69%	65%	64%
BANK	**74%**	**100%**	63%	**73%**	**74%**	**75%**	**71%**	59%	**92%**	**75%**	**75%**	**87%**	39%	63%	62%	67%	59%	64%
BRES	**73%**	63%	**100%**	**83%**	**81%**	**78%**	60%	66%	69%	56%	**78%**	56%	44%	31%	59%	62%	52%	41%
CHEM	**83%**	**73%**	**83%**	**100%**	**85%**	**82%**	**72%**	67%	**72%**	**74%**	**82%**	70%	51%	45%	69%	65%	57%	57%
CONS	**78%**	**74%**	**81%**	**85%**	**100%**	**90%**	**72%**	66%	**75%**	**76%**	**89%**	64%	54%	39%	67%	66%	63%	64%
CYCL	**75%**	**75%**	**78%**	**82%**	**90%**	**100%**	67%	64%	**79%**	70%	**87%**	63%	58%	43%	67%	**70%**	63%	56%
CNYL	**73%**	**71%**	60%	**72%**	**72%**	67%	**100%**	55%	69%	**75%**	69%	**74%**	41%	58%	**73%**	53%	59%	**71%**
ENGY	55%	59%	66%	67%	66%	64%	55%	**100%**	59%	59%	59%	54%	28%	43%	55%	43%	32%	46%
FISV	**73%**	**92%**	69%	**72%**	**75%**	**79%**	69%	59%	**100%**	**75%**	**73%**	**85%**	39%	61%	58%	64%	57%	58%
FBEV	**71%**	**75%**	56%	**74%**	**76%**	70%	**75%**	59%	**75%**	**100%**	62%	**74%**	27%	63%	61%	40%	41%	66%
INDS	**79%**	**75%**	**78%**	**82%**	**89%**	**87%**	69%	59%	**73%**	62%	**100%**	65%	**72%**	38%	68%	**82%**	**77%**	67%
INSU	**72%**	**87%**	56%	70%	64%	63%	**74%**	54%	**85%**	**74%**	65%	**100%**	36%	67%	61%	60%	56%	68%
MEDA	46%	39%	44%	51%	54%	58%	41%	28%	39%	27%	**72%**	36%	**100%**	21%	42%	**75%**	**77%**	57%
PHRM	43%	63%	31%	45%	39%	43%	58%	43%	61%	63%	38%	67%	21%	**100%**	43%	35%	37%	58%
RETL	68%	62%	59%	69%	67%	67%	**73%**	55%	58%	61%	68%	61%	42%	43%	**100%**	52%	53%	57%
TECH	69%	67%	62%	65%	66%	**70%**	53%	43%	64%	40%	**82%**	60%	**75%**	35%	52%	**100%**	**81%**	55%
TELE	65%	59%	52%	57%	63%	63%	59%	32%	57%	41%	**77%**	56%	**77%**	37%	53%	**81%**	**100%**	70%
UTLY	64%	64%	41%	57%	64%	56%	**71%**	46%	58%	66%	67%	68%	57%	58%	57%	55%	70%	**100%**

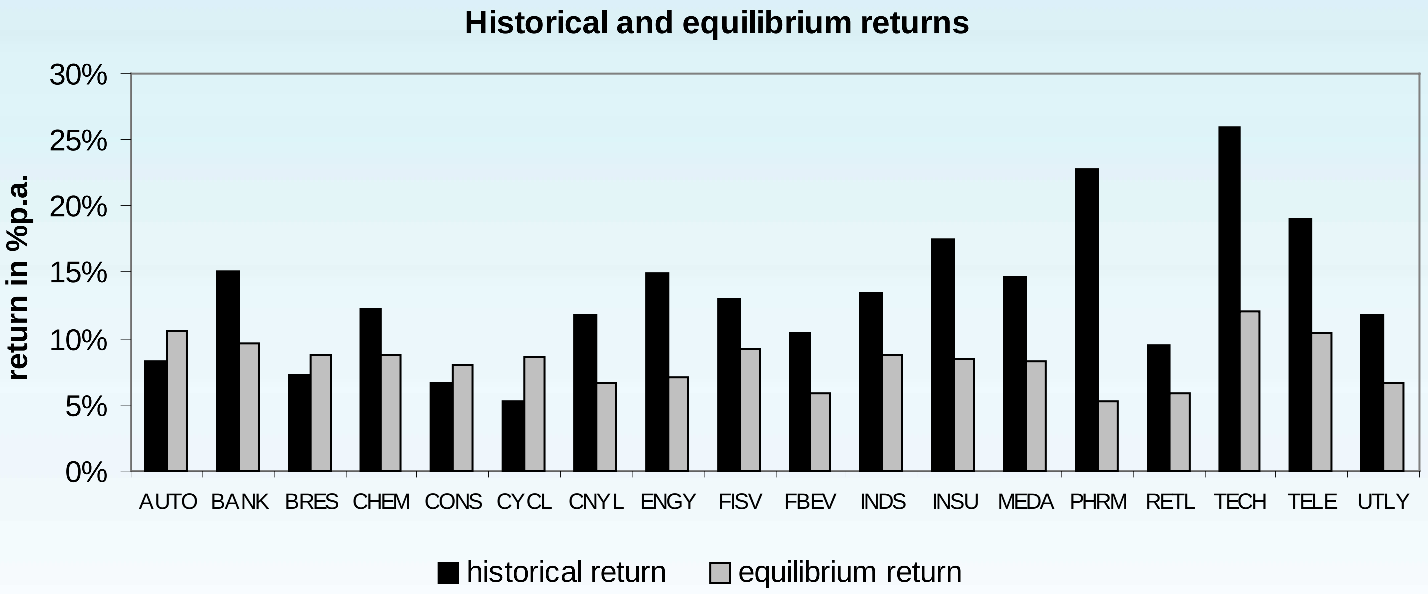
- Covariance matrix via $\Omega_{ij} = \sigma_i \sigma_j \rho_{ij}$.

Black-Litterman Approach - the equilibrium returns

BL-starting point: Equilibrium - implicit - returns of market portfolio

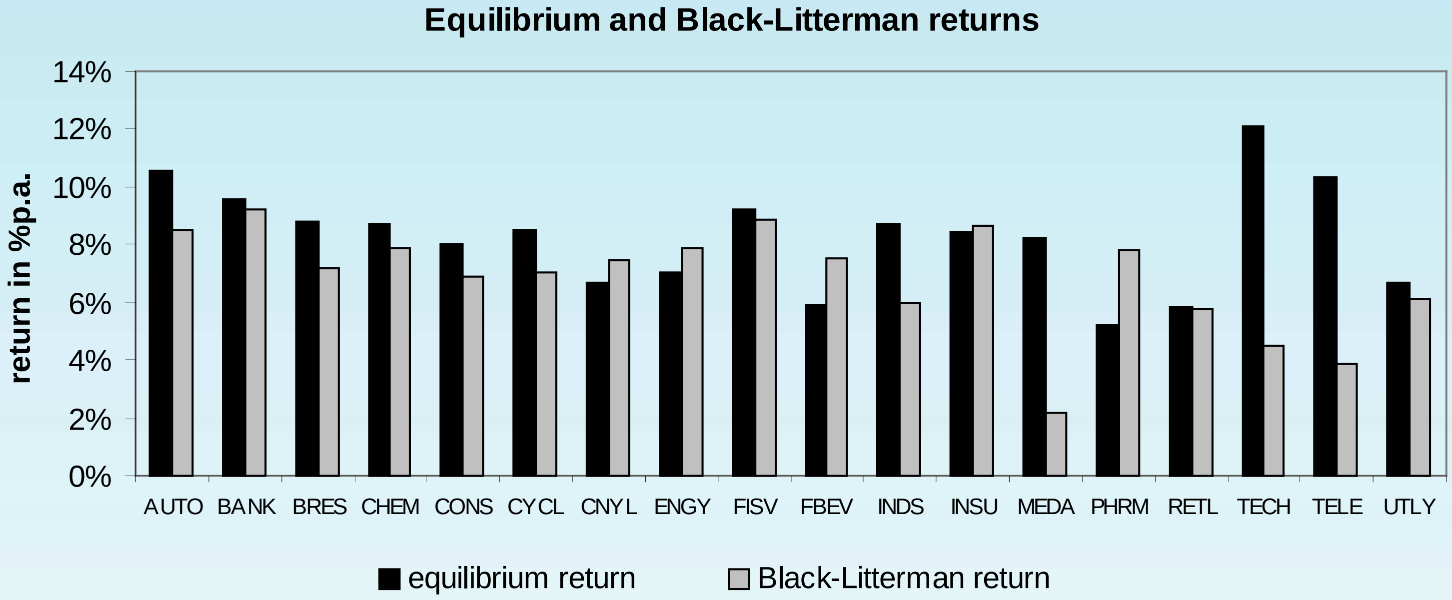
- “inverse optimization“ yields **equilibrium returns**

$$R_{Equilibrium} = (\tau\,\Omega)\cdot w_{Equilibrium} \qquad \text{where} \quad w_{Equilibrium} = w_{Market\ Portfolio}$$



Black-Litterman Approach - the BL returns

From equilibrium returns to Black-Litterman returns (Views as given)

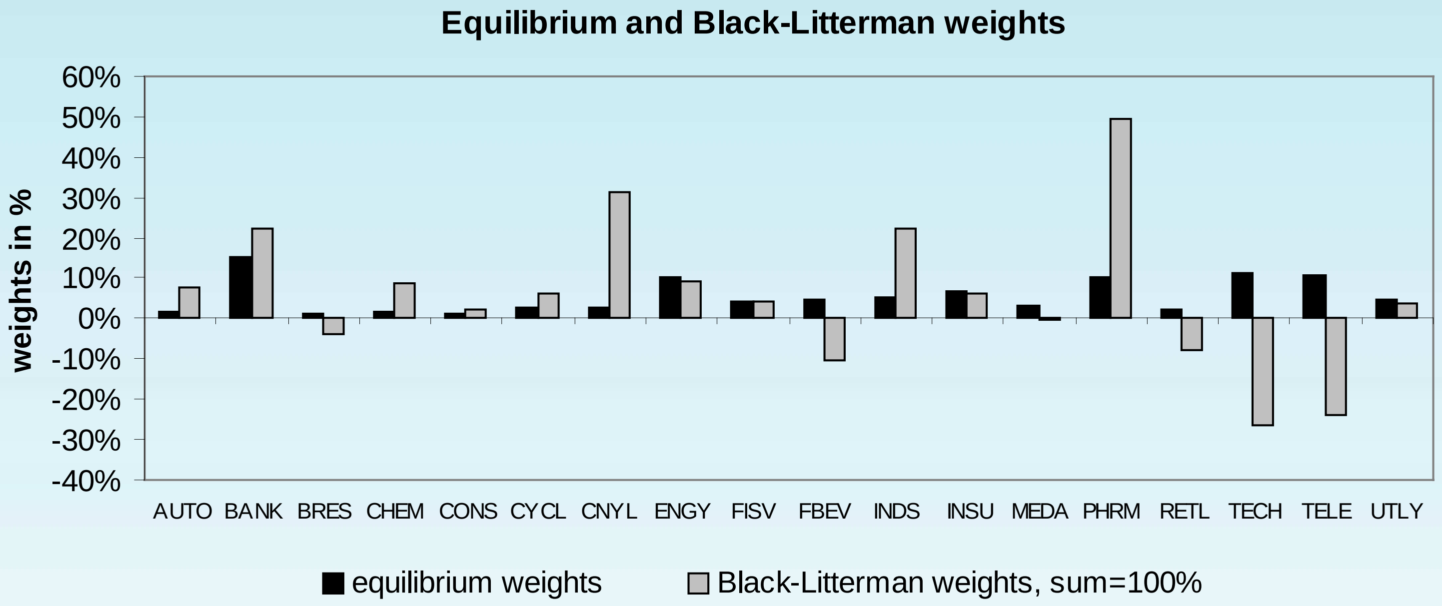


Having implemented the Views... :

- BL return expectations significantly lowered for TECH und TELE.
- BL return expectation higher in PHRM but lower in INDS (still okay because the *relative* View „... better than TELE und TECH“ remains intact !)
- For CNYL, the expected return shifts from 6.66% to 7.48% (90% confidence in View 7,5%).
- Example: MEDA (correlated by 75% to TECH, 77% to TELE) has significantly lower return.

Black-Litterman Approach - the BL weights

Comparing equilibrium weights (market cap.) and Black-Litterman weights

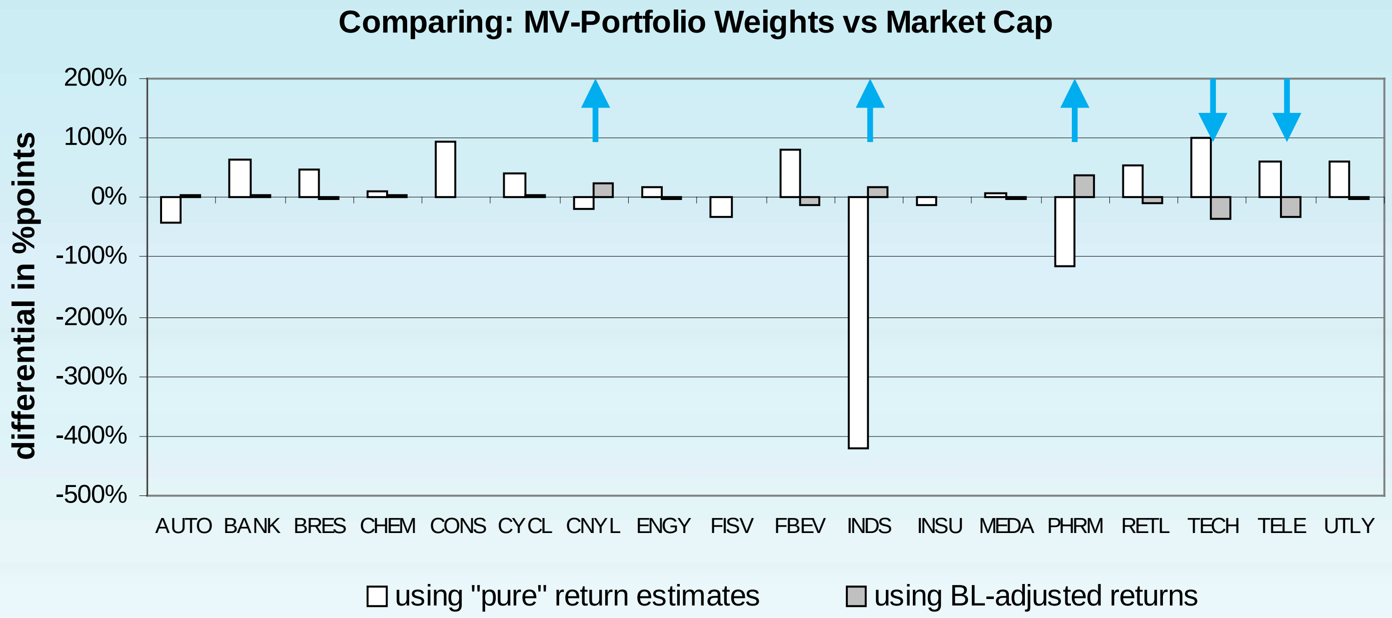


Using the BL returns (note: strong confidence in Views of 90%):

- Significant weight reduction for TELE and TECH (also, e.g., for MEDA)
- Significant weight increase for INDS and PHRM
- Significantly increased weight for CNYL due to higher return expectation (+0.84%pts)

Black-Litterman Approach vs straight MV

Given the return scenario (Views up or down,↑↓), the revised portfolio structure clearly benefits from the BL-enhanced optimization process.



- **Straight MV optimization** - which is a naiv approach in terms of „c.p.“ return estimates - yields extreme and unreliable changes in portfolio weights
- **The BL-adjusted return input for MV** stabilizes the weights, leading to a reliable and intuitively sound new portfolio structure.

Black-Litterman Approach - constraints I

Calculation of weights s.t. constraints

- **In general: Use mean/variance optimizer with constraints**
- **No constraints:**

$$w = \frac{1}{\gamma}\,\Omega^{-1}\cdot R$$

- **“Budget constraint“, i.e. sum of weights = 100%** (I = *1-Vector*)**:**

$$w = \frac{\Omega^{-1}\,I}{I^{\mathrm{T}}\,\Omega^{-1}\,I} + \frac{1}{\gamma}\,\Omega^{-1}\cdot\left(R - \frac{I^{\mathrm{T}}\,\Omega^{-1}\,R}{I^{\mathrm{T}}\,\Omega^{-1}\,I}\,I\right)$$

Black-Litterman Approach - constraints II

Calculation of weights s.t. constraints

- Additional constraint: **Tracking Error** (w_{act} = active weights)

$$TE^2 = w_{act}{}^T \cdot \Omega \cdot w_{act}$$

- Additional constraint: **Portfolio-BETA** („directional risk in the portfolio“)

$$\beta_P = \sum_{i=1}^{\#Assets} w_i \beta_i$$

- Additional constraint: „**No short positions**“

$$w_i \geq 0 \quad \forall\, i = 1..\#\,\text{Assets}$$

(see, e.g., paper of K. Iordanidis); in Excel: requires additional calculations & solver constraints

Black-Litterman Approach - weights

Remark on treating weights w.r.t. absolute / relative Views

- The sum of portfolio weights has to add up to **100%**.
- **Purely *absolute* Views** are translated into independent *long* and *short* portfolios, thus causing portfolio weights to deviate from 100%. Therefore, normalization of weights is recommended.
- **Purely *relative* Views** are translated into weight-balanced *long* and *short* portfolios, so that portfolio weights still sum up to 100%.
- The use **of *absolute and relative* Views** again leads to portfolio weights deviating from 100% Therefore, again, normalization of weights is recommended.
- The normalization to 100% has to be included in the optimization process as a constraint.
- *Note that - unfortunately - the use of constraints is contra-intuitive for BL-adjusted allocations.*

Black-Litterman Approach

Numerical example on treating weights w.r.t. absolute / relative Views

- The example is based on the View scenario given on slide 24.

- **Purely *absolute* View:** Weights add up to 110%, with all weights unchanged except for the asset (sector) CNYL under view (weight increases by 10%pts.).

  Normalization to 100% leads to weight changes in *all* positions - as it should be!

  (Note: Lowering the expected return of CNYL to, e.g., 5,5% yields a total portfolio weight of only 86%. Again, weight normalization is recommended, spreading for the -14%pts across *all* weights.)

- **Purely *relative* Views:** Weights add up to 100%, with the weights of the viewed assets just offsetting the *long* and *short* positions.

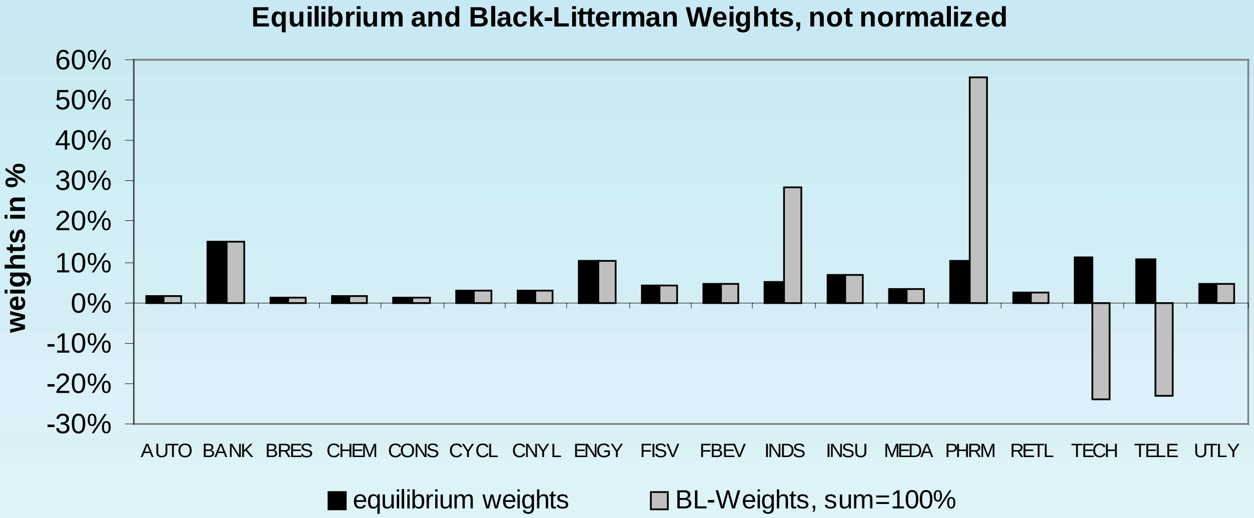
- ***Absolute <u>and</u> relative* Views:** Weights add up to 129%, with *long* and *short* positions for the viewed assets as intuitively expected and the unviewed assets‘ weights remaining unchanged. Normalization to 100% consequently leads to weight changes in *all* positions - as it should be!

Black-Litterman Approach

(cont.) non-normalized weights

Purely relative Views

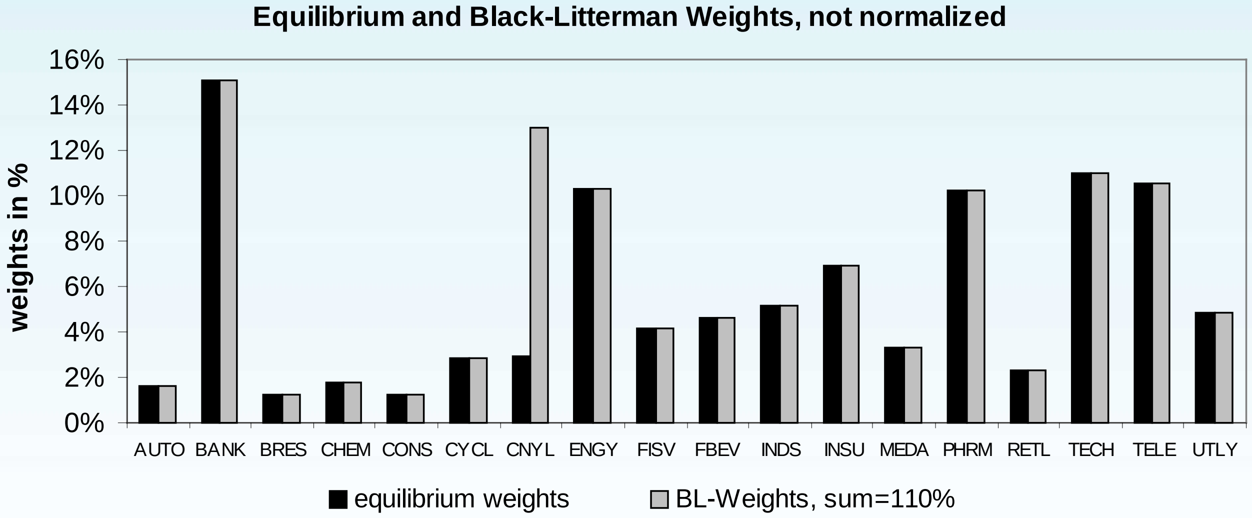
Σ weights = 100%,
no normalization required



Purely absolute Views

Σ weights = 110%,

normalization recommended



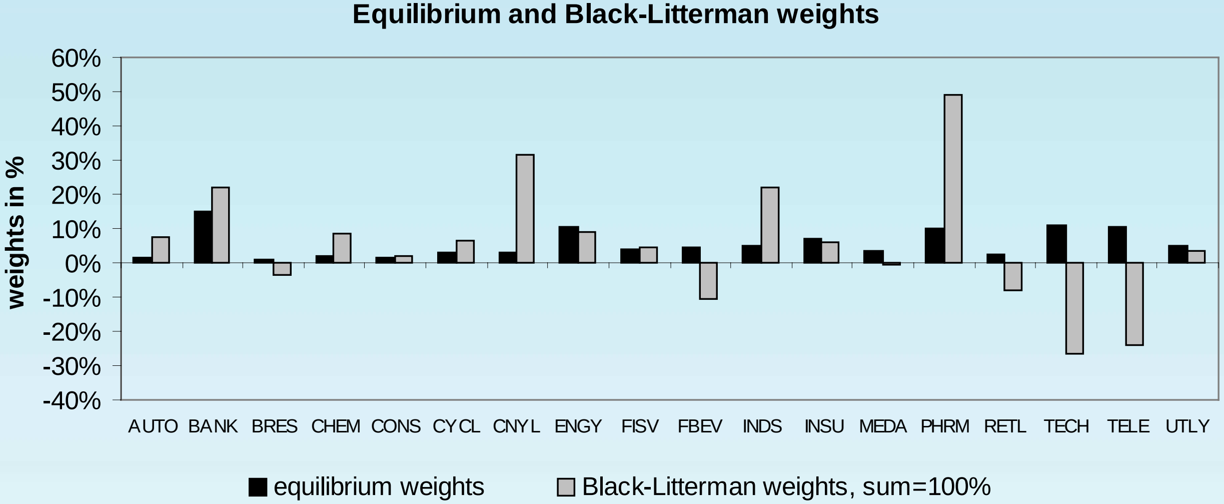
Black-Litterman Approach

Numerical example (cont.)

Relative and absolute Views

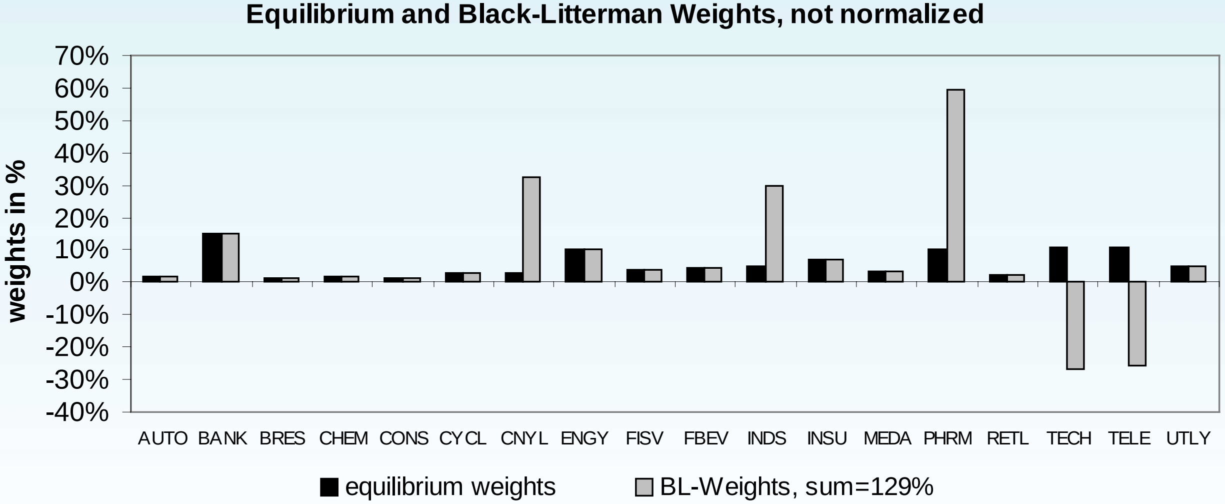
- weights normalized

  Σ weights = 100%



- weights not normalized

  Σ weights = 129%



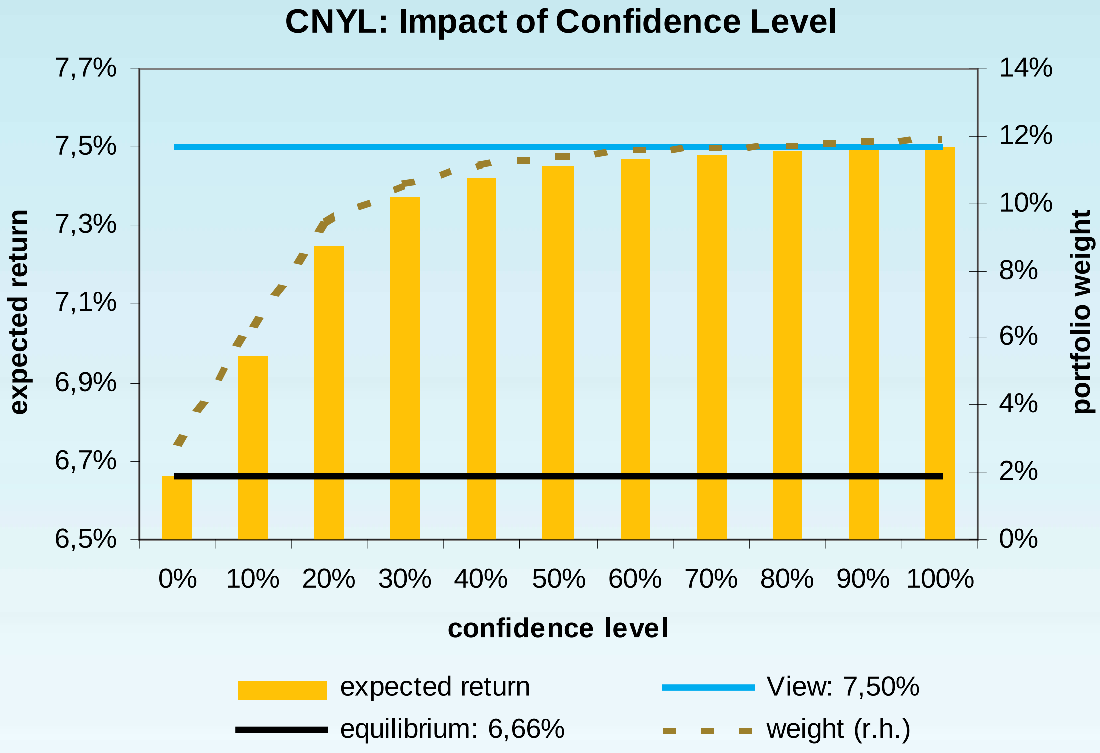
Example: DJ STOXX

Black-Litterman Approach - confidence and CNYL

Influence of degree of confidence on BL-returns and BL-weights, I

Focus on asset „CNYL“ only:

- Equilibrium return = 6.66%,
- Equilibrium weight = 2.90%
- View on return: 7.5% ± 1.5% (equivalent to a range of 6 - 9%)



Observations:

- Low confidence: → equilibrium return
- High confidence: Asymptotic approach to the View value of 7.5%.
- Limit: At a confidence level of 100%, BL fully accepts the strong view of 7.5%.
- Weights: from 2.9% (= market cap, due to confidence of 0% a *no view*-case), up to 12% (overweighting due to the *strong view* confidence of 100%).

Black-Litterman Approach - weights of CNYL I

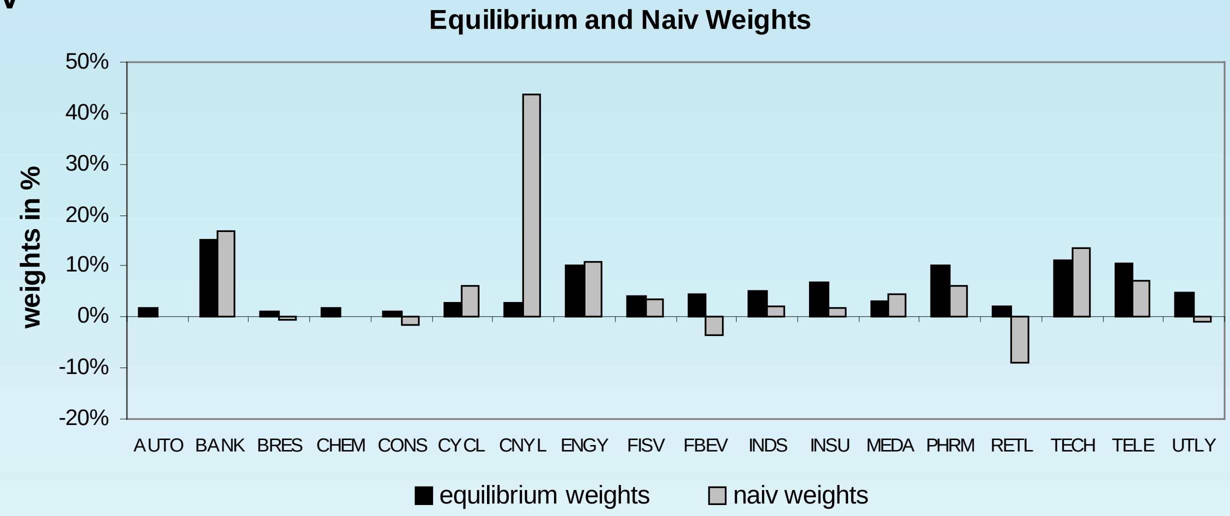
BL compared to straight MV

CNYL

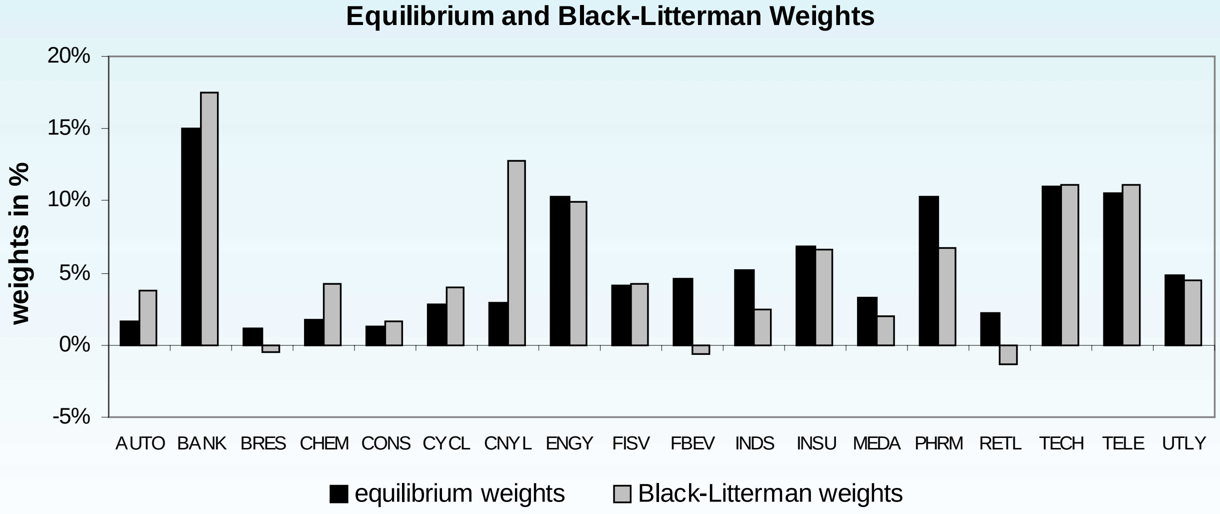
View on Return:
from 6.6% up to 7.5%
(with strong confidence)

Result

- Realistic weight changes in BL
- Volatile weight scenario in straight MV approach



Equilibrium and Naiv Weights



Equilibrium and Black-Litterman Weights

Black-Litterman Approach - weights of CNYL II

BL compared to straight MV

CNYL

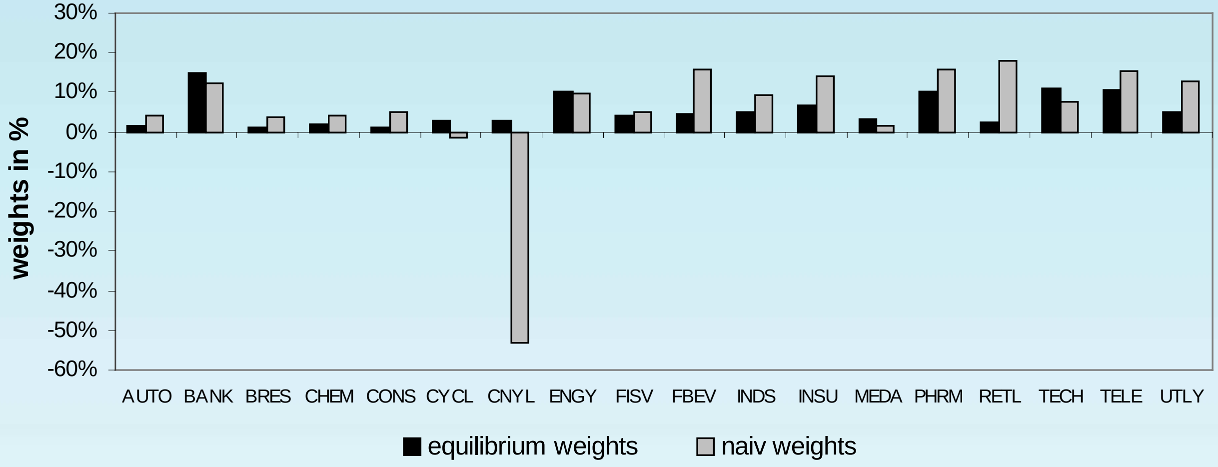
View on Return:

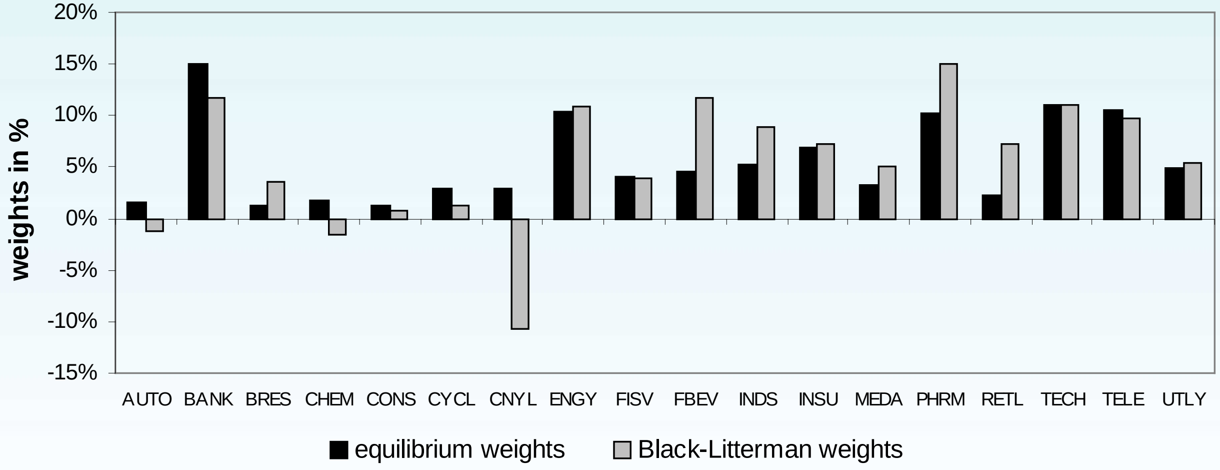
from 6.6% down to 5.5%

(with strong confidence)

Result

- Realistic weight changes in BL
- Volatile weight scenario in straight MV approach


Equilibrium and Naiv Weights


Equilibrium and Black-Litterman Weights

Black-Litterman Approach - confidence and weights

Influence of degree of confidence on BL-returns and BL-weights , **II**

Behavior of asset weights:

- Complete portfolio, 18 sectors

Observations:

- Low degrees of confidence:

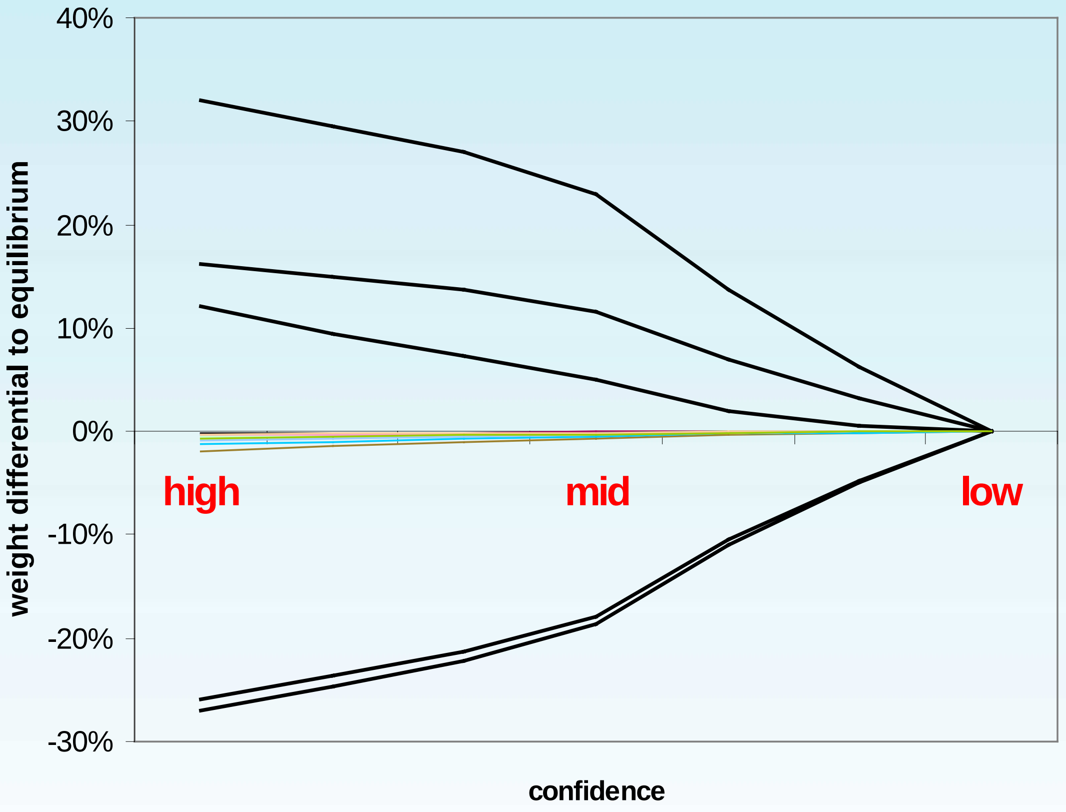
  BL-weights are close to weights in equilibrium (=market cap‘s).

- Higher degree of confidence:

  Weights approach equilibrium values on either underweighting (*short*) or overweighting (*long*) path.

- Most significant weight changes for the assets under View.

Sensitivity of weights on degree of confidence

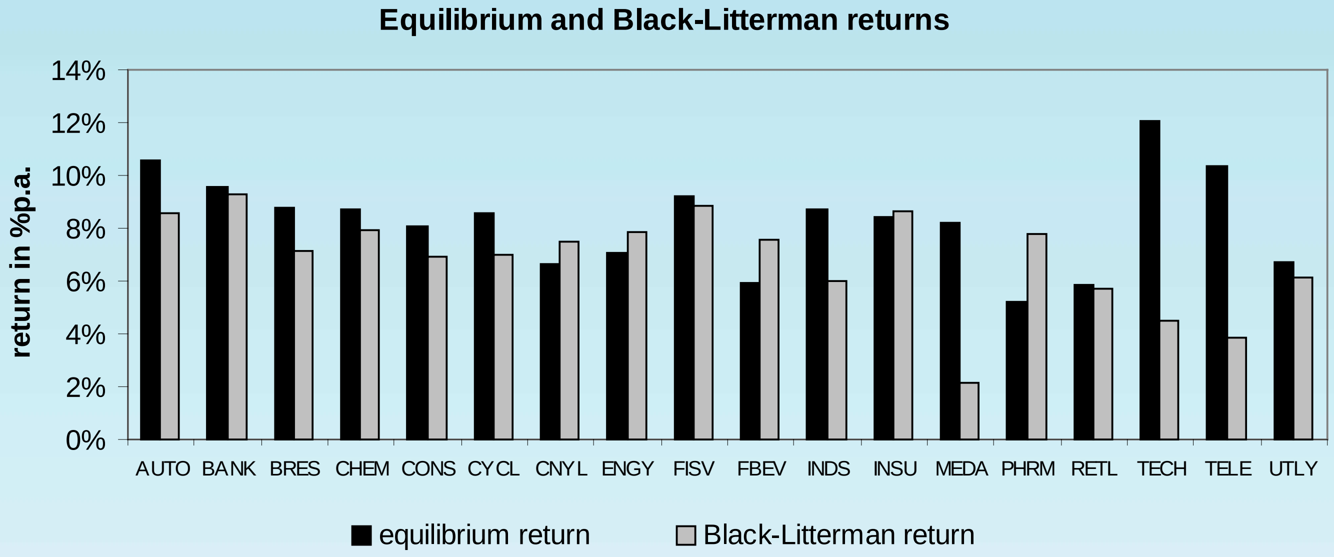


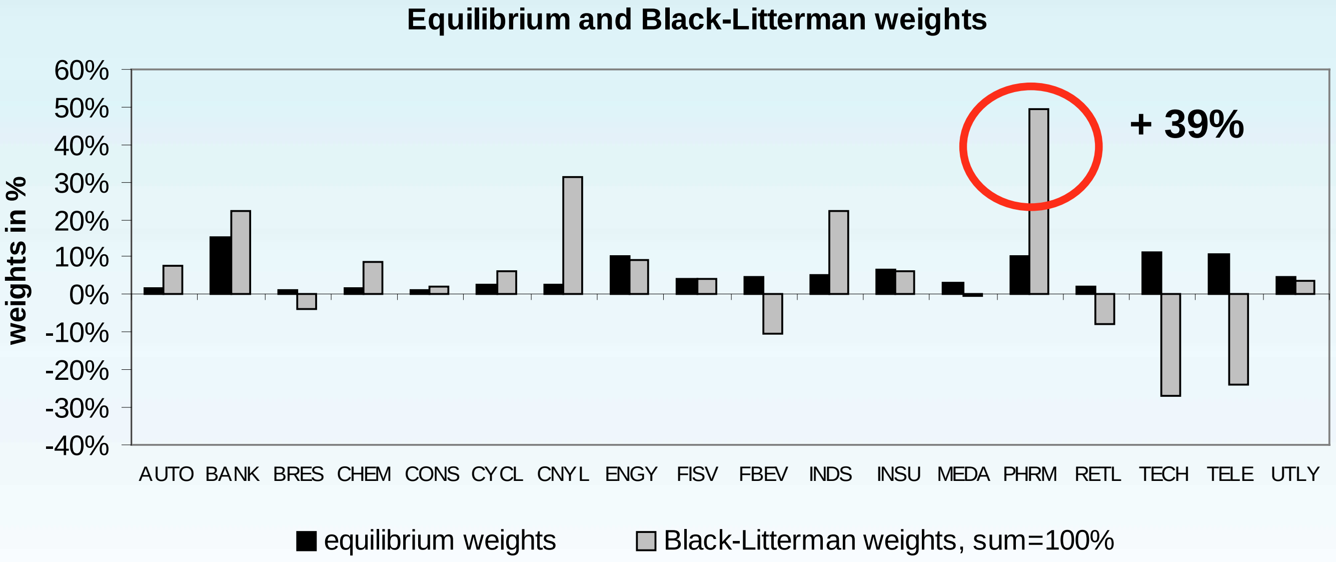
Example: DJ STOXX

Black-Litterman Approach

Strong confidence

- Large changes in weights due to the “strong views“





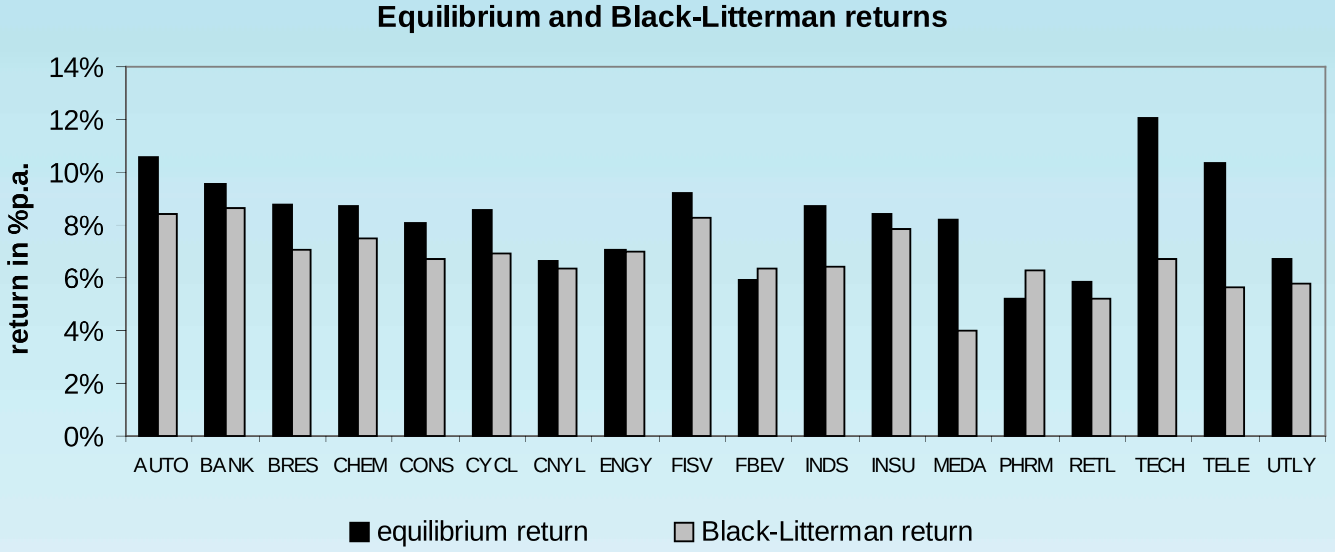
+ 39%

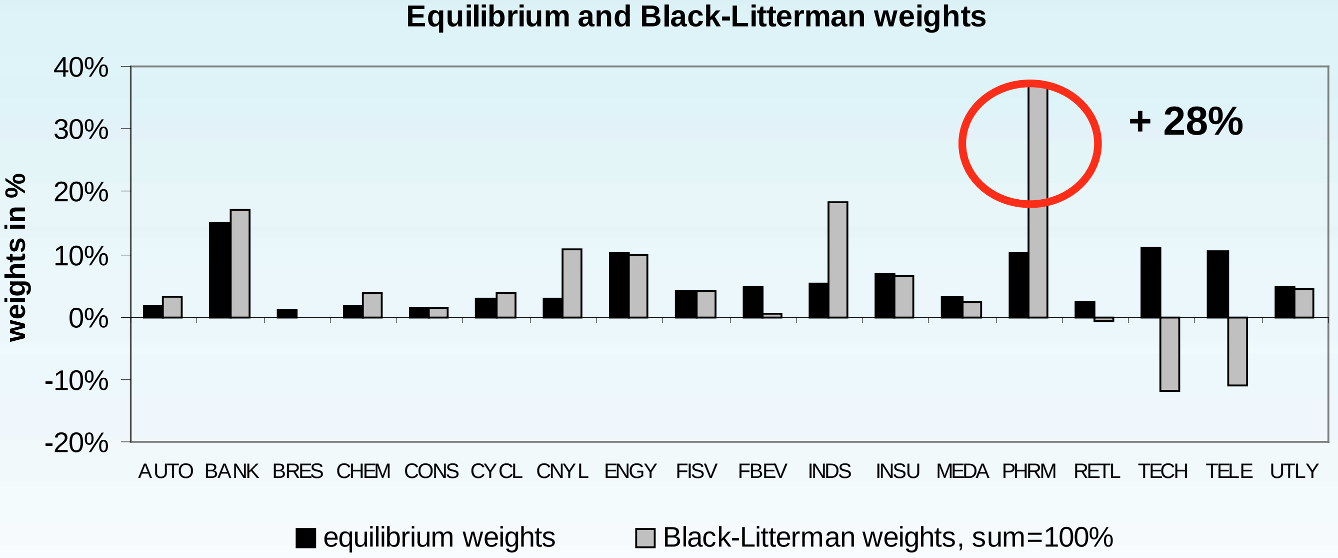
Example: DJ STOXX

Black-Litterman Approach

Mid confidence

- Moderate changes in weights

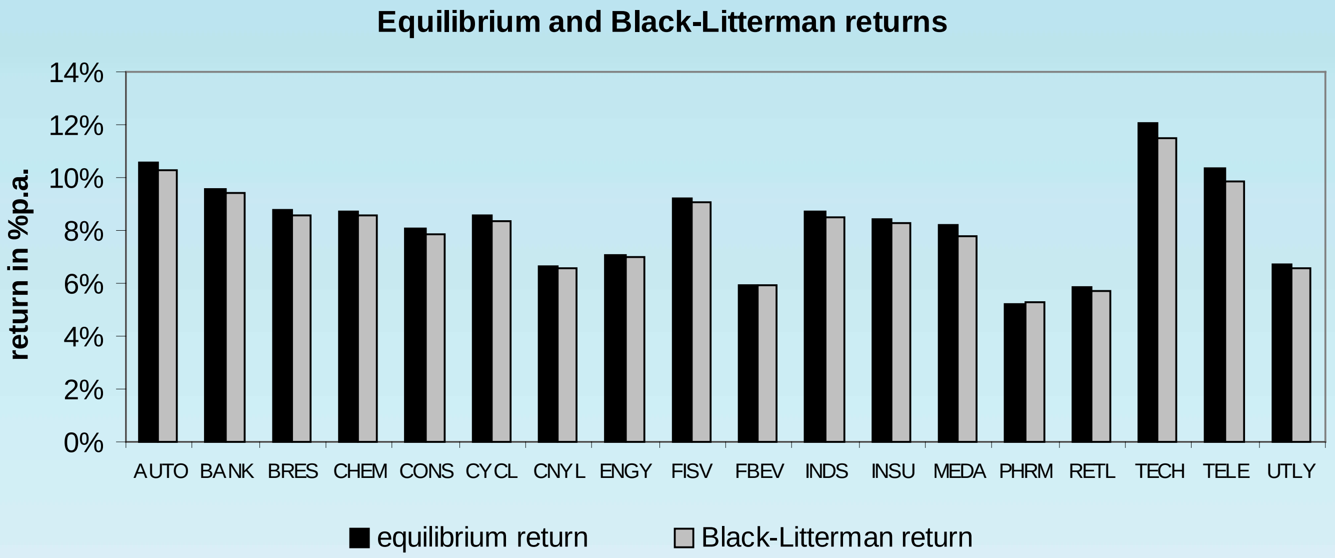




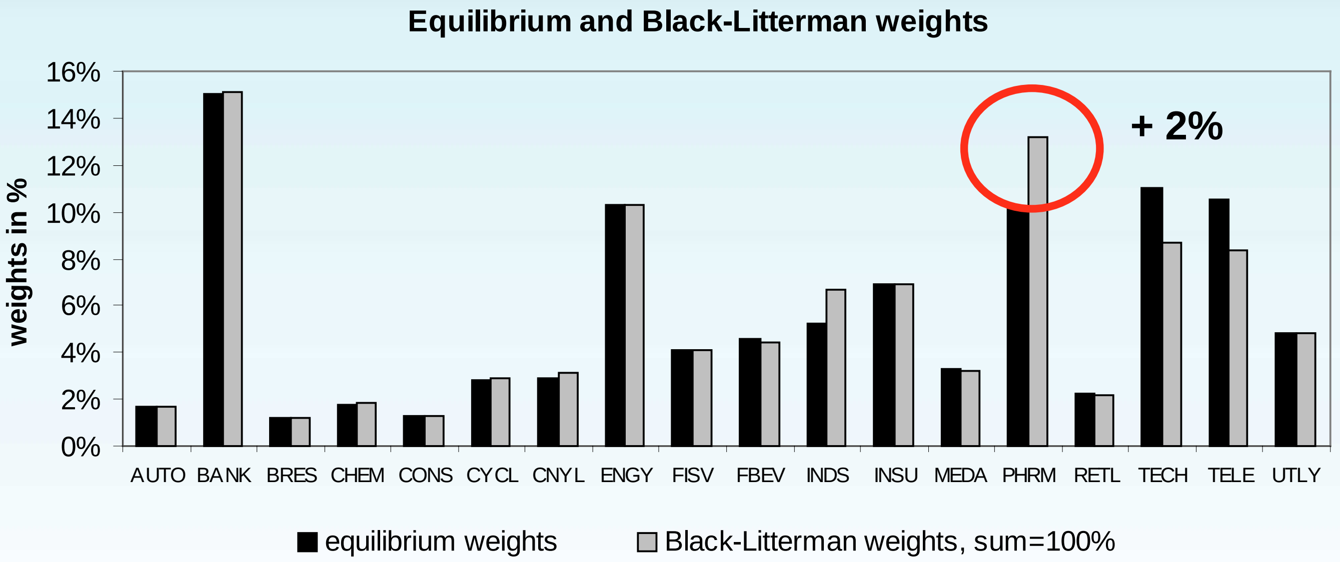
Black-Litterman Approach

Poor confidence

- Weights stay close to equilibrium weights



Equilibrium and Black-Litterman returns



Equilibrium and Black-Litterman weights

+ 2%

Black-Litterman Approach - Conclusion I

Traditional „Straight MV“ vs „BL plus MV“ approach

	straight MV	**Black-Litterman → MV**
Return estimates:		
	o required for each asset	**required only for selected assets**
	o assumed as certain	**degree of confidence**
	o absolute return figures	**absolute or relative Views**
	o c.p.	**consistent**
Reference return:		
	o none	**equilibrium returns**

Black-Litterman Approach - Conclusion II

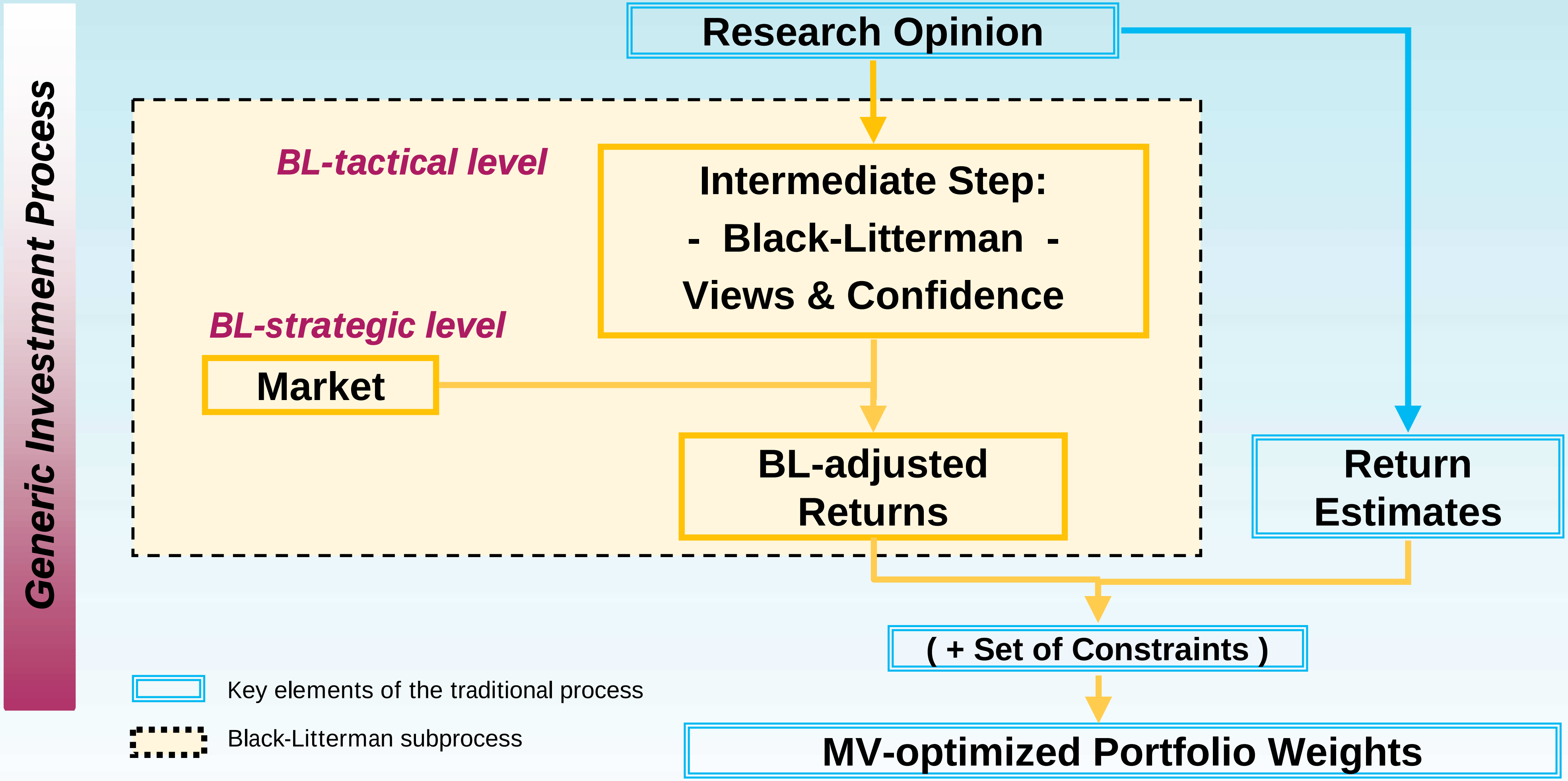
Traditional „Straight MV“ vs „BL plus MV“ approach

- **MV-optimized Portfolios:**

	straight MV	**Black-Litterman → MV**
o	extreme asset weights	**reliable asset weights**
o	changes in return estimates	
	⇒ huge weight fluctuations	**⇒ moderate weight changes**
o	portfolios unreliable	**consistent structure**
		„intuitively reasonable“
o	MV-results hardly accepted	**higher acceptance**
o	reflecting c.p. opinions	**„correlated Views“**

Black-Litterman Approach - Conclusion III

BL as a building-block of an enhanced asset allocation process

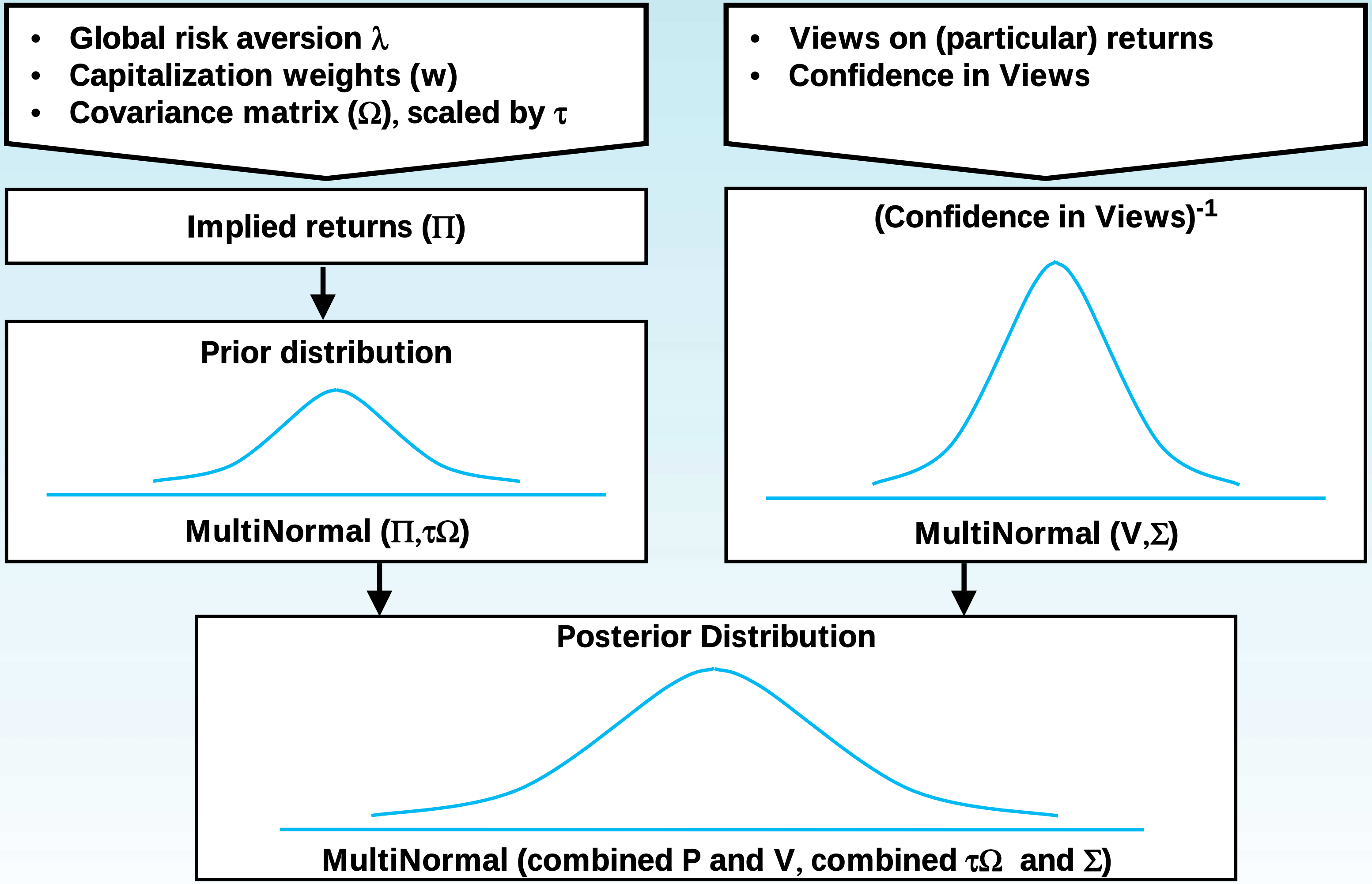


▭ Key elements of the traditional process

⬚ Black-Litterman subprocess

Black-Litterman Approach - Conclusion IV

... in a more formal way



- Global risk aversion λ
- Capitalization weights (w)
- Covariance matrix (Ω), scaled by τ

- Views on (particular) returns
- Confidence in Views

Implied returns (Π)

(Confidence in Views)$^{-1}$

Prior distribution

MultiNormal (Π,$\tau\Omega$)

MultiNormal (V,Σ)

Posterior Distribution

MultiNormal (combined P and V, combined $\tau\Omega$ and Σ)

Layout inspired by
K.Iordanidis

Black-Litterman Approach - more insights

Suggestions for further reading...

- Black F. and Litterman R.: *Global Portfolio Optimization*, Fin.Analysts Journal, Sep.1992
- Black F. and Litterman R.: *Asset Allocation: Combining Investor Views with Market Equilibrium*, Goldman-Sachs, Fixed Income Research, Sep.1990
- Zimmermann H., Drobetz W. and Oertmann P.: *Global Asset Allocation: New Methods and Applications, publ. by Wiley & Sons,* Nov.2002
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- Bevan A. and Winkelmann K.: *Using the BL Global Asset Allocation Model: Three Years of Practical Experience,* Goldman-Sachs, Fixed Income Research, Jun.1998

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- Drobetz T.: *Einsatz des BL-Verfahrens in der Asset Allocation*, Working paper, Mar.2002
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- Zimmermann H. et al.: *Einsatz des Black-Litterman-Verfahrens in der Asset Allocation*, in „Handbuch Asset Allocation“, Editors: Dichtl, Schlenger u. Kleeberg, Uhlenbruch-Verlag, 2002.